

- Previous Lecture:
 - Developing algorithms
 - Nested loops
- Today's Lecture:
 - Review
 - Finite vs. Infinite
 - Introduction to vectors, vectorized code
- Announcements:
 - Discussion this week in UP B7 lab
 - Prelim I: 9/23(R) 7:30-9pm. Location Statler Aud.
 - Review sessions: 9/21(T) 5-6:30pm, 2/22(W) 5-6:30pm. Locations TBA. They're *optional*—attend one if you wish.

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Rational approximation of π

- $\pi = 3.141592653589793\dots$
- Can be closely approximated by fractions, e.g., $\pi \approx 22/7$
- Rational number: a quotient of two integers
- Approximate π as p/q where p and q are positive integers $\leq M$
- Start with a straight forward solution:
 - Get M from user
 - Calculate quotient p/q for all combinations of p and q
 - Pick best quotient \rightarrow smallest error

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```
% Rational approximation of pi

M = input('Enter M: ');
% Best q, p, and error so far
qBest=1; pBest=1;
err_pq = abs(pBest/qBest - pi);
% Check all possible denominators
for q = 1:M
    % Find best numerator for this q
    for p = 1:M % Check all possible p

    end

end

myPi = pBest/qBest;
```

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Analyze the program for efficiency

- See Eg3_1 and FasterEg3_1 in the book

```
for a = 1:n
    disp('alpha')
    for b = 1:m
        disp('beta')
    end
end
```

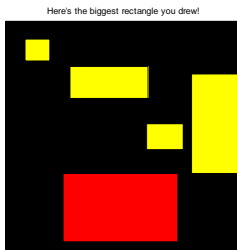
How many times are “alpha” and “beta” displayed?

- A: n, m
- B: m, n
- C: n, n+m
- D: n, n*m
- E: m*n, m

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Find the biggest rectangle



Here's the biggest rectangle you drew!

- Draw 5 rectangles that the user specifies using mouse clicks
- Color the biggest one red

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The savvy programmer...

- Learns useful **programming patterns** and use them where appropriate
- Seeks inspiration by **working through test data “by hand”**
 - Asks, “What am I doing?” at each step
 - Sets up a variable for each piece of information maintained when working the problem by hand
- **Decomposes** the problem into manageable subtasks
 - Refines the solution **iteratively**, solving simpler subproblems first
- Remembers to check the problem's boundary conditions
- Validates the solution (program) by trying it on test data

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Xeno's Paradox

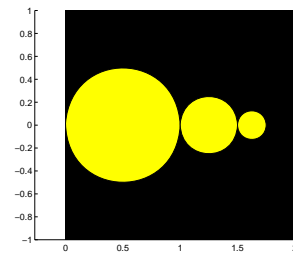
- A wall is two feet away
- Take steps that repeatedly halve the remaining distance
- You never reach the wall because the distance traveled after n steps =

$$1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$$

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Example: "Xeno" disks



Draw a sequence of 20 disks where the $(k+1)$ th disk has a diameter that is half that of the k th disk.

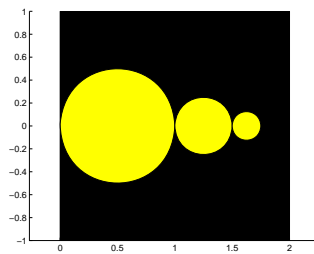
The disks are tangent to each other and have centers on the x-axis.

First disk has diameter 1 and center $(1/2, 0)$.

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Example: "Xeno" disks



What do you need to keep track of?

- Diameter (d)
- Position
Left tangent point (x)

Disk	x	d
1	0	1
2	$0+1$	$1/2$
3	$0+1+1/2$	$1/4$

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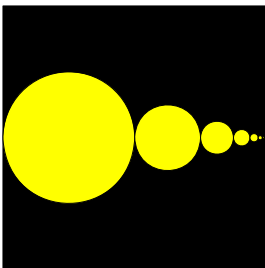
% Xeno Disks

```
DrawRect(0,-1,2,2,'k')
% Draw 20 Xeno disks
```

```
for k= 1:20
```

```
end
```

Here's the output... Shouldn't there be 20 disks?



The "screen" is an array of dots called pixels.

Disks smaller than the dots don't show up.

The 20th disk has radius < .000001

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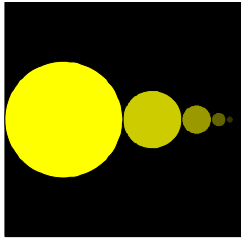
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End of Review Material for Prelim 1

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Fading Xeno disks



- First disk is yellow
- Last disk is black (invisible)
- Interpolate the color in between

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Color is a 3-vector, sometimes called the RGB values

- Any color is a mix of red, green, and blue
- Example:
`colr = [0.4 0.6 0]`
- Each component is a real value in [0,1]
- `[0 0 0]` is black
- `[1 1 1]` is white

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```
% Draw n fading Xeno disks
d= 1;
x= 0; % Left tangent point
yellow= [1 1 0];
black= [0 0 0];
for k= 1:n
    % Compute color of kth disk

    % Draw kth disk
    DrawDisk(x+d/2, 0, d/2, _____)
    x= x+d;
    d= d/2;
end
```

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Example: 3 disks fading from yellow to black

```
r= 1; % radius of disk
yellow= [1 1 0];
black = [0 0 0];

% Left disk yellow, at x=1
DrawDisk(1,0,r,yellow)
% Right disk black, at x=5
DrawDisk(5,0,r,black)

% Middle disk with average color, at x=3
colr= 0.5*yellow + 0.5*black;
DrawDisk(3,0,r,colr)
```



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Linear interpolation

x	g(x)
:	:
9	110
10	118
10.25	?
10.50	?
10.75	?
11	126
12	134
:	:

$$g(10.5) = \frac{1}{2} g(11) + \frac{1}{2} g(10)$$



$$g(10.25) = \frac{1}{4} \cdot g(11) + \frac{3}{4} \cdot g(10)$$

$$g(10.50) = \frac{2}{4} \cdot g(11) + \frac{2}{4} \cdot g(10)$$

$$g(10.75) = \frac{3}{4} \cdot g(11) + \frac{1}{4} \cdot g(10)$$

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Rows of Xeno disks



```
for y = __:__:__
```

```
Code to draw one
row of Xeno disks
at some y-coordinate
```

```
end
```

Be careful with "initializations"

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