# Rational approximation of $\pi$

- $\pi$ = 3.141592653589793…
- Can be closely approximated by fractions,

  e.g., $\pi \approx 22/7$
- Rational number: a quotient of two integers
- Approximate $\pi$ as p/q where p and q are positive integers ≤M
- Start with a straight forward solution:
  - Get M from user
  - Calculate quotient p/q for all combinations of p and q
  - Pick best quotient → smallest error

```
% Rational approximation of pi

M = input(‘Enter M: ’);



% Check all possible denominators
```

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% Rational approximation of pi

M = input('Enter M: ');



% Check all possible denominators
for q = 1:M

```

For current q find best numerator p…
Check all possible numerators

```


end
```

```
% Rational approximation of pi

M = input('Enter M: ');



% Check all possible denominators
for q = 1:M
  % Find best numerator for this q

  for p = 1:M  % Check all possible p



  end



end
```

```
% Rational approximation of pi

M = input('Enter M: ');
% Best q, p, and error so far
qBest=1;  pBest=1;
err_pq = abs(pBest/qBest - pi);
% Check all possible denominators
for q = 1:M
  % Find best numerator for this q

  for p = 1:M  % Check all possible p



  end



end
myPi = pBest/qBest;
```

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% Check all possible denominators
for q = 1:M
  % Find best numerator for this q
  p0=1;  e0=abs(p0/q - pi);  % best p & error so far
  for p = 1:M  % Check all possible p



  end



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% Check all possible denominators
for q = 1:M
  % Find best numerator for this q
  p0=1;  e0=abs(p0/q - pi);  % best p & error so far
  for p = 1:M  % Check all possible p
    if abs(p/q - pi) < e0  % new best numerator found
      p0=p;  e0 = abs(p/q - pi);
    end
  end
```

Now we have the best p for this q.
Is the quotient at this q best among all previous q's?

```
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for q = 1:M
  % Find best numerator for this q
  p0=1;  e0=abs(p0/q - pi);  % best p & error so far
  for p = 1:M  % Check all possible p
    if abs(p/q - pi) < e0  % new best numerator found
      p0=p;  e0 = abs(p/q - pi);
    end
  end
  % Is best quotient for this q is best over all?
  if e0 < err_pq
    pBest=p0;  qBest=q; err_pq=e0;
  end
end
myPi = pBest/qBest;
```