Announcements

- P6 due today at 11pm
- Final exam: 12/10 (Fri) 9am at Barton West (indoor field)
- Please fill out course evaluation on-line (hosted by College of Engineering, see "Exercise15")
- Please fill out evaluation on iRobot Create Simulator on CMS. It is worth 1 project point (to make up for any lost project point)!
- Regular office/consulting hours end today. "Study Break" hours start next week.
- Review Session: 12/8 Wednesday 1-2:30pm, UP B17
- Pick up any paper from consultants (prelim, regrade results) during consulting hours. Everything will be shredded afterwards.
- Read announcements on course website!

- Previous Lecture:
  - Efficiency
  - Recursion
- Today's Lecture:
  - Recursion review
  - A model to quantify importance: Google "Page Rank"

Quantifying Importance

How do you rank web pages for importance given that you know the link structure of the Web, i.e., the in-links and out-links for each web page?

A related question:

How does a deleted or added link on a webpage affect its "rank"?

Background

Index all the pages on the Web from 1 to n. (n is around ten billion.)

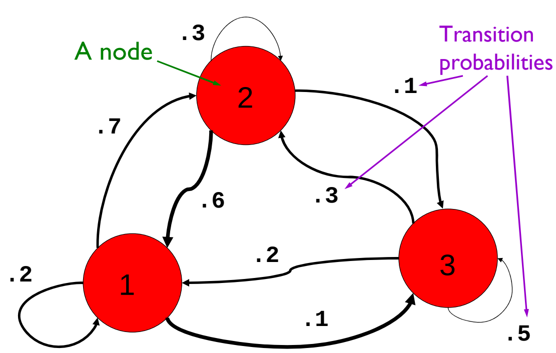
The PageRank algorithm orders these pages from "most important" to "least important."

It does this by analyzing links, not content.

Key ideas

- There is a random web surfer—a special random walk
- The surfer has some random "surfing" behavior—a transition probability matrix
- The transition probability matrix comes from the link structure of the web—a connectivity matrix
- Applying the transition probability matrix → Page Rank

A 3-node network with specified transition probabilities



A special random walk

Suppose there are a 1000 people on each node.

At the sound of a whistle they hop to another node in accordance with the "outbound" probabilities.

For now we assume we know these probabilities. Later we will see how to get them.

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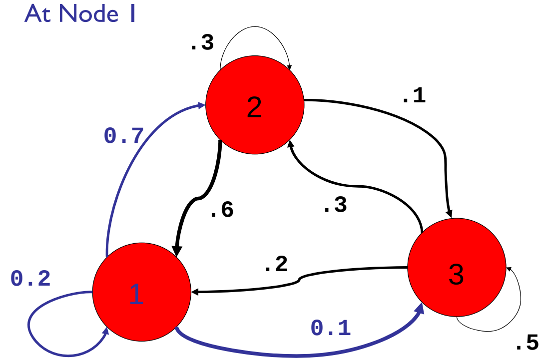
At Node 1



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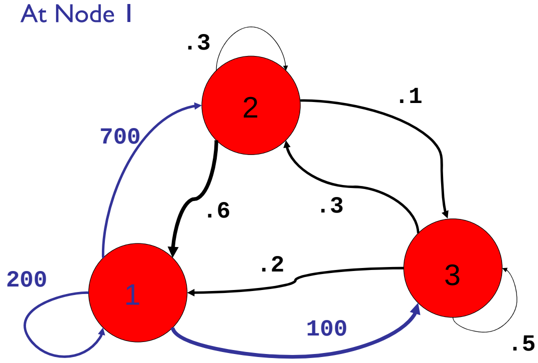
At Node 1



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State Vector:
describes the state at each node at a specific time

T=0	T=1	T=2
1000	1000	1120
1000	1300	1300
1000	700	580

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After 100 iterations

	T=99	T=100
Node 1	1142.85	1142.85
Node 2	1357.14	1357.14
Node 3	500.00	500.00

Appears to reach a steady state

Call this the stationary vector

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Formula for the new state vector

P

.2	.6	.2
.7	.3	.3
.1	.1	.5

P(i,j) is probability of hopping to node i from node j

```
w(1) = P(1,1)*v(1) + P(1,2)*v(2) + P(1,3)*v(3)
w(2) = P(2,1)*v(1) + P(2,2)*v(2) + P(2,3)*v(3)
w(3) = P(3,1)*v(1) + P(3,2)*v(2) + P(3,3)*v(3)
```

v is the old state vector
w is the updated state vector

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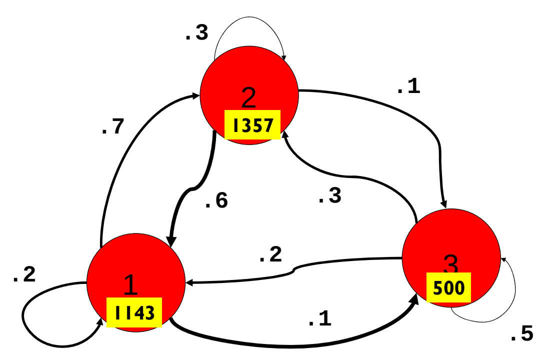
The general case

```
function w = Update(P,v)
% Update state vector v based on transition
% probability matrix P to give state vector w
n = length(v);
w = zeros(n,1);
for i=1:n
   for j=1:n
      w(i) = w(i) + P(i,j)*v(j);
   end
end
```


To obtain the stationary vector…

```
function [w,err]= StatVec(P,v,tol,kMax)
% Iterate to get stationary vector w
w = Update(P,v);
err = max(abs(w-v));
k = 1;
while k<kMax && err>tol
      v = w;
      w = Update(P,v);
      err = max(abs(w-v));
      k = k+1;
end
```


Stationary vector indicates importance: 2 1 3



A random walk on the web

Repeat:
You are on a webpage.
There are m outlinks, so choose one at random.
Click on the link.

Random island hopping

Repeat:
You are on an island.
According to the transitional probabilities,
go to another island.

Use the link structure of the web to figure out the transitional probabilities!

(Assume no dead ends for now; we deal with them later.)

Connectivity Matrix

G

0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	1	0	1
0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0

`G(i,j)` is **1** if there is a link on page `j` to page `i`.

(I.e., you can get to `i` from `j`.)

Connectivity Matrix

G

0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	1	0	1
0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0

Transition Probability Matrix

derived from Connectivity Matrix

P

0	0	0	0	0	0	?	?
?	0	0	?	0	0	0	0
?	0	?	0	0	?	0	?
0	0	0	0	?	0	0	0
?	0	?	0	0	0	0	?
0	0	?	0	0	0	0	?
0	0	?	0	0	0	0	0
0	?	0	?	0	0	0	0

Connectivity (G) → Transition Probability (P)

```
[n,n] = size(G);
P = zeros(n,n);
for j=1:n
   P(:,j) = G(:,j)/sum(G(:,j));
end
```


To obtain the stationary vector…

```
function [w,err]= StatVec(P,v,tol,kMax)
% Iterate to get stationary vector w
w = Update(P,v);
err = max(abs(w-v));
k = 1;
while k<kMax && err>tol
      v = w;
      w = Update(P,v);
      err = max(abs(w-v));
      k = k+1;
end
```


Stationary vector represents how "popular" the pages are → PageRank

statVec	sorted	idx	pR
0.5723	0.8911	6	4
0.8206	0.8206	2	2
0.7876	0.7876	3	3
0.2609	0.5723	1	6
0.2064	0.4100	8	8
0.8911	0.2609	4	1
0.2429	0.2429	7	7
0.4100	0.2064	5	5


```
[sorted, idx] = sort(-statVec);
for k= 1:length(statVec)
   j = idx(k);  % index of kth largest
   pR(j) = k;
end
```

statVec	sorted	idx	pR
0.5723	-0.8911	6	4
0.8206	-0.8206	2	2
0.7876	-0.7876	3	3
0.2609	-0.5723	1	6
0.2064	-0.4100	8	8
0.8911	-0.2609	4	1
0.2429	-0.2429	7	7
0.4100	-0.2064	5	5

The random walk idea gets transitional probabilities from connectivity. Can modify the random walk to deal with dead ends.

```
Repeat:
     You are on a webpage.
     If there are no outlinks
        Pick a random page and go there.
     else
        Flip an unfair coin.
        if heads
           Click on a random outlink and go there.
        else
           Pick a random page and go there.
        end
     end
```

In practice, an unfair coin with prob .85 heads works well.

This results in a different transitional probability matrix.

What we learned…

- Develop/implement algorithms for problems
- Develop programming skills
  - Design, implement, document, test, and debug
- Programming "tool bag"
  - Control flow (if-else; loops)
  - Functions for reducing redundancy
  - Data structures
  - Graphics
  - File handling

What we learned… (cont'd)

- Applications and concepts
  - Image and sound
  - Sorting and searching—you should know the algorithms covered
  - Divide-and-conquer strategies
  - Approximation and error
  - Simulation
  - Computational effort and efficiency

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Final Exam

- Mon 12/10, 9-11:30am, Barton West
- Covers entire course, but emphasizes material after Prelim 3
- Closed-book exam, no calculators
- Bring student ID card

- Check for announcements on webpage:
  - Study break office/consulting hours
  - Review questions
  - List of potentially useful functions

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