

Lecture 26

Advanced Sorting

Announcements for This Lecture

Exam & Assignments

- **Prelim, Dec. 4 at 7:30**
 - See webpage for rooms
 - Last call for conflicts!
- Review **Dec. 1 at 7:30**
 - Check Ed/Website for **room**
- A6 still being graded
 - Will be ready next class
- Keep working on A7
 - Labs today are open OH

Optional Videos

- **ALL** all are now posted
 - **Lesson 28** for **today**
 - **Lesson 29** is next week



Recall Our Problem

- Both insertion, selection sort are **nested loops**
 - **Outer loop** over each element to sort
 - **Inner loop** to put next element in place
 - Each loop is n steps. $n \times n = n^2$
- To do better we must *eliminate* a loop
 - But how do we do that?
 - What is like a loop? **Recursion!**
 - First need an *intermediate* algorithm

The Partition Algorithm

- Given a list segment $b[h..k]$ with some value x in $b[h]$:

Start: b

x	?
-----	---

- Swap elements of $b[h..k]$ to get this answer

Goal: b

$\leq x$	x	$\geq x$
----------	-----	----------

change: b

3	5	4	1	6	2	3	8	1
-----	---	---	---	---	---	---	---	---

into b

1	2	1	3	5	4	6	3	8
---	---	---	-----	---	---	---	---	---

or b

1	2	3	1	3	4	5	6	8
---	---	---	---	-----	---	---	---	---

- x is called the **pivot value**
 - x is not a program variable
 - denotes value initially in $b[h]$

Designing the Partition Algorithm

- Given a list $b[h..k]$ with some value x in $b[h]$:

Start: b

h		k
x	?	

- Swap elements of $b[h..k]$ to get this answer

Goal: b

h		i	$i+1$	k
$\leq x$	x	$\geq x$		

In-Progress: b

h		i	j	k
$\leq x$	x	?	$\geq x$	

Indices b, h important!
Might partition only part

Implementating the Partition Algorithm

```
def partition(b, h, k):  
    """Partition list b[h..k] around a pivot x = b[h]"""  
    i = h; j = k+1; x = b[h]  
  
    while i < j-1:  
        if b[i+1] >= x:  
            # Move to end of block.  
            swap(b,i+1,j-1)  
            j = j - 1  
        else: # b[i+1] < x  
            swap(b,i,i+1)  
            i = i + 1  
  
    return i
```

partition(b,h,k), not partition(b[h:k+1])
Remember, slicing always copies the list!
We want to partition the **original** list

Partition Algorithm Implementation

```
def partition(b, h, k):  
    """Partition list b[h..k] around a pivot x = b[h]"""  
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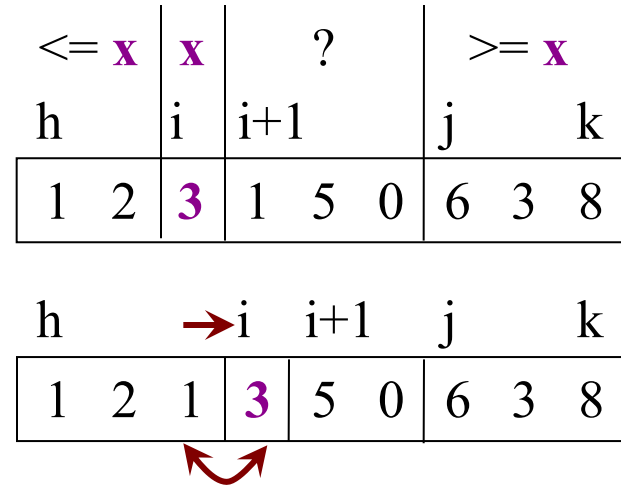
<= x		x	?	>= x		
h		i	i+1	j		k
1	2	3	1 5 0	6	3	8

Partition Algorithm Implementation

```
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<= x		x	?	>= x		
h		i	i+1	j		k
1	2	3	1 5 0	6	3	8

h		→ i		i+1	j	k
1	2	1	3	5 0	6	3 8



h		i	j	← k		
1	2	1	3	0	5	6 3 8



Partition Algorithm Implementation

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<= x		x	?			>= x		
h		i	i+1			j		k
1	2	3	1	5	0	6	3	8

h		→ i		i+1	j		k	
1	2	1	3	5	0	6	3	8



h		i		j ←		k		
1	2	1	3	0	5	6	3	8



h		→ i		j	k			
1	2	1	0	3	5	6	3	8

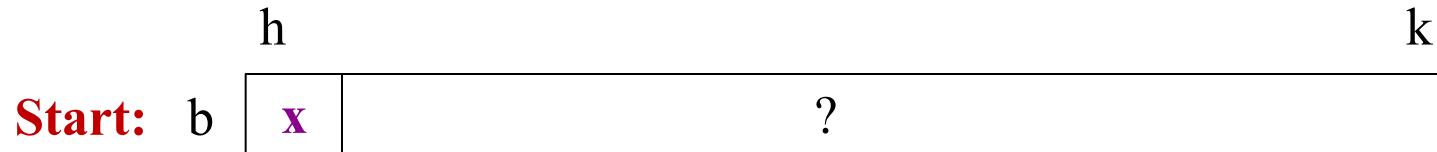


Why is this Useful?

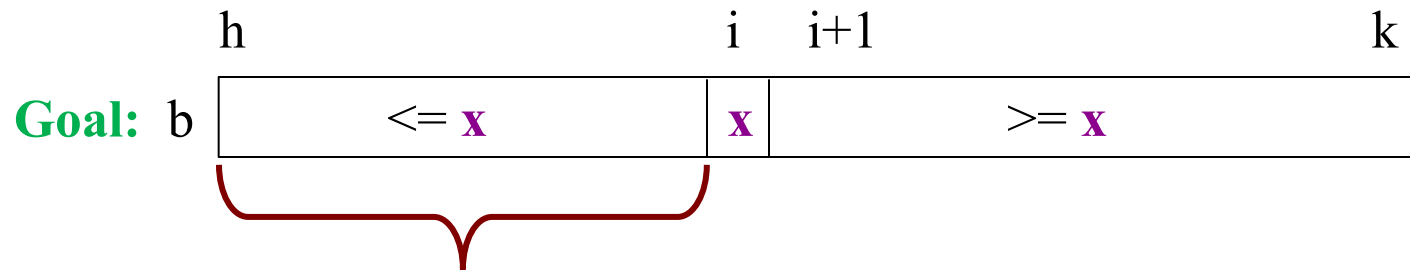
- Will use this algorithm to replace inner loop
 - The inner loop cost us n swaps every time
- Can this reduce the number of swaps?
 - Worst case is $k-h$ swaps
 - This is n if partitioning the whole list
 - But less if only partitioning part
- **Idea:** Break up list and partition only part?
 - This is **Divide-and-Conquer!**

Sorting with Partitions

- Given a list segment $b[h..k]$ with some value x in $b[h]$:



- Swap elements of $b[h..k]$ to get this answer



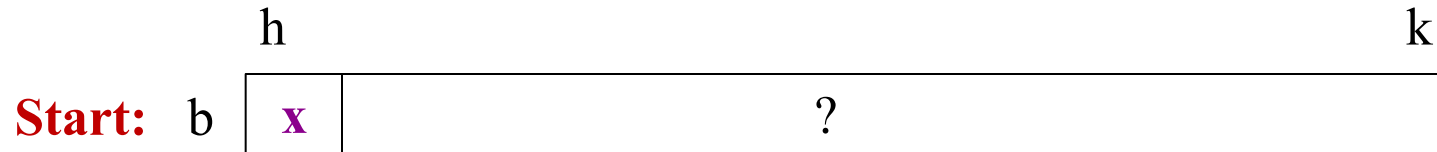
Partition Recursively

Recursive partitions = sorting

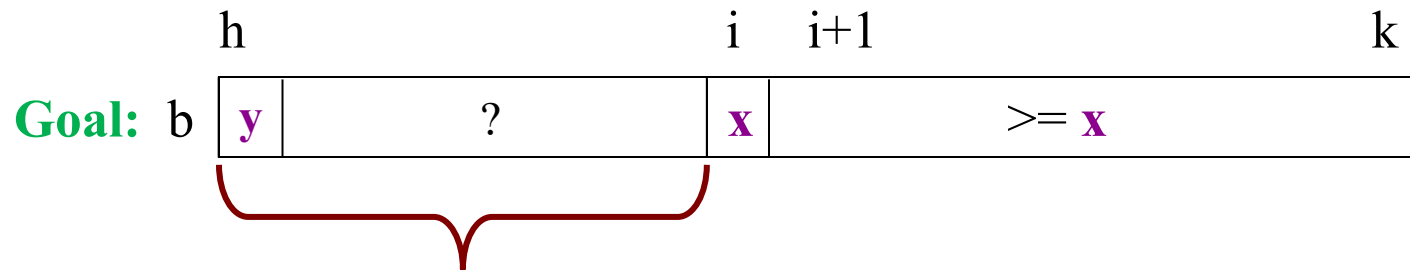
- Called **QuickSort** (why???)
- Popular, fast sorting technique

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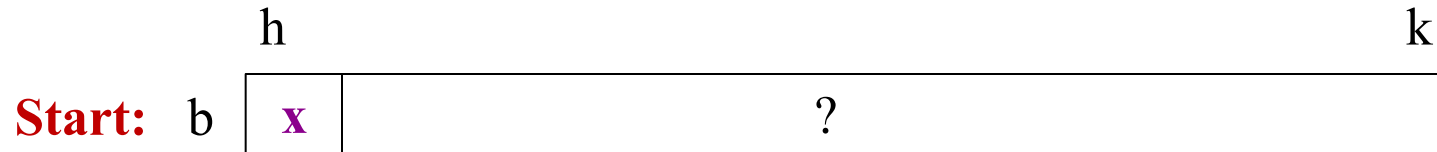
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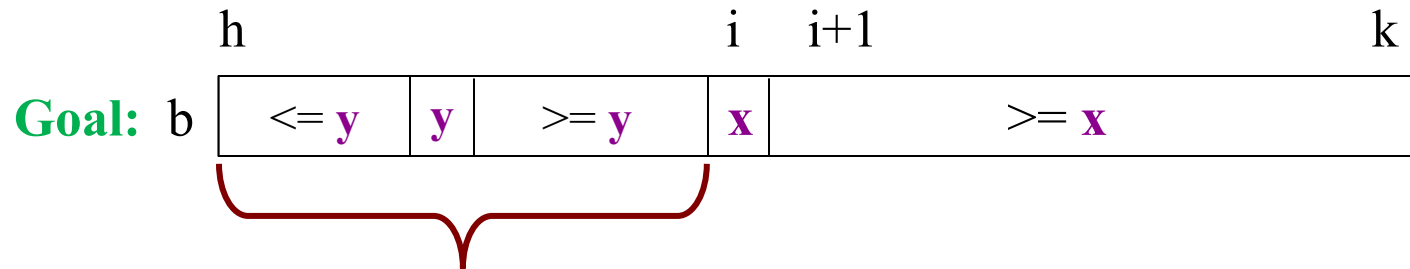
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Sorting with Partitions

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Partition Recursively

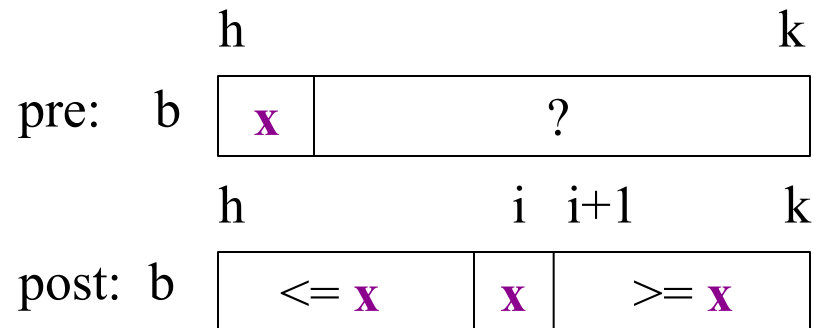
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- Called **QuickSort** (why???)
- Popular, fast sorting technique

QuickSort

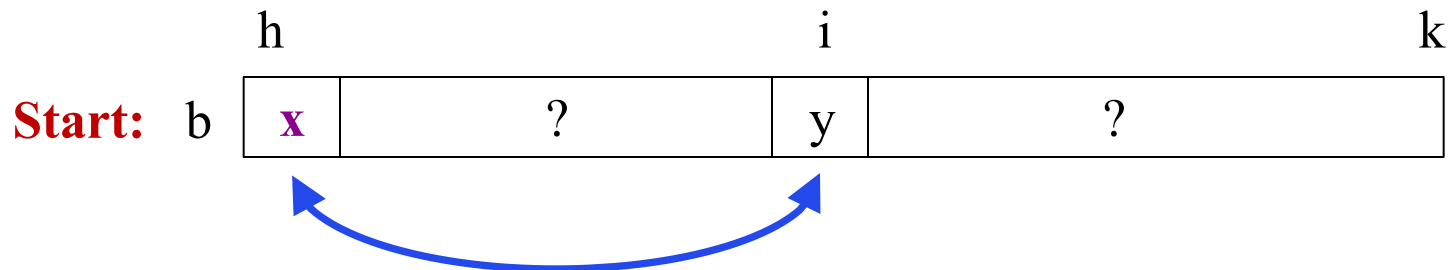
```
def quick_sort(b, h, k):  
    """Sort the array fragment b[h..k]"""  
    if b[h..k] has fewer than 2 elements:  
        return  
    j = partition(b, h, k)  
    # b[h..j-1] <= b[j] <= b[j+1..k]  
    # Sort b[h..j-1] and b[j+1..k]  
    quick_sort(b, h, j-1)  
    quick_sort(b, j+1, k)
```

- **Worst Case:**
array already sorted
 - Or almost sorted
 - n^2 in that case
- **Average Case:**
array is scrambled
 - $n \log n$ in that case
 - Best sorting time!



So Does that Solve It?

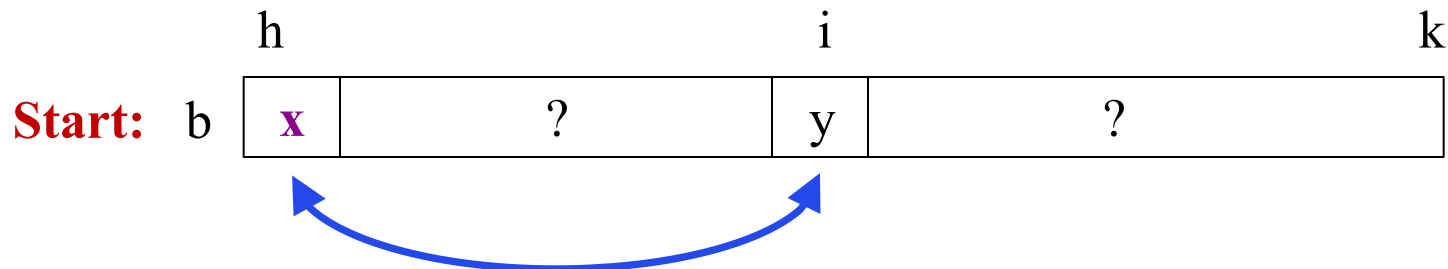
- Worst case still seems bad! Still n^2
 - But only happens in small number of cases
 - Just happens that case is common (already sorted)
- Can greatly reduce issue with randomization
 - Swap start with random element in list
 - Now pivot is random and already sorted unlikely



So Does that Solve It?

- Worst case still seems bad! Still n^2
 - But only happens in small number of cases
 - Just happens in already sorted (or reverse sorted) arrays
- Can greatly reduce the worst case (randomization)
 - Swap start and random element
 - Now pivot is random and already sorted unlikely

Makes it “good enough”
for most applications



Can We Do Better?

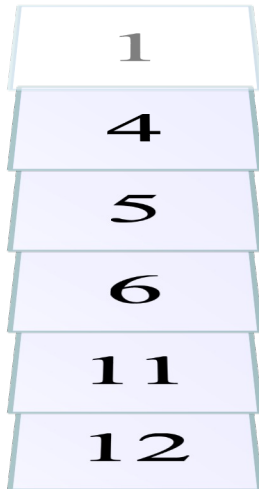
- Recursion seems to be the solution
 - Partitioned the list into two halves
 - Recursively sorted each half
- How about a traditional **divide-and-conquer**?
 - **Divide** the list into two halves
 - **Recursively sort** the two halves
 - **Combine** the two sort halves
- How do we do the last step?

Combining Two Sorted Lists

1
4
5
6
11
12

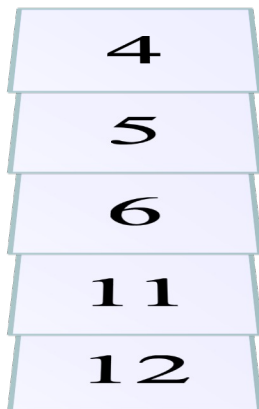
2
3
7
8
9
10

Combining Two Sorted Lists



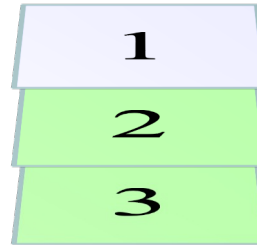
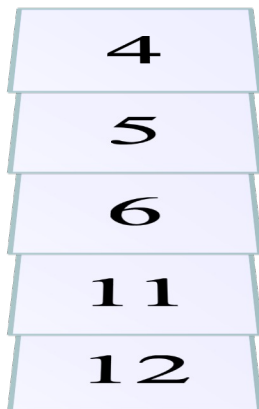
Pick from list
with the least

Combining Two Sorted Lists



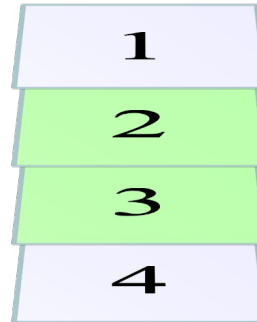
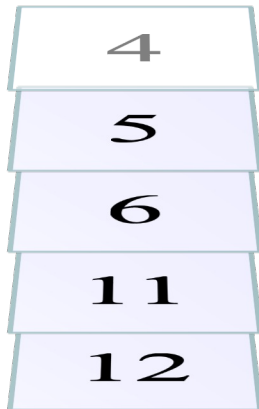
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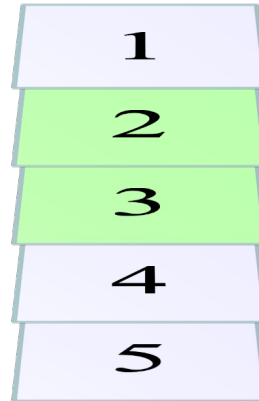
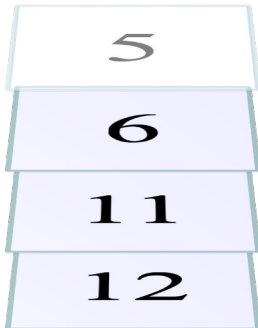
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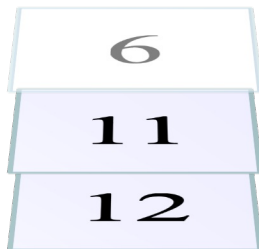
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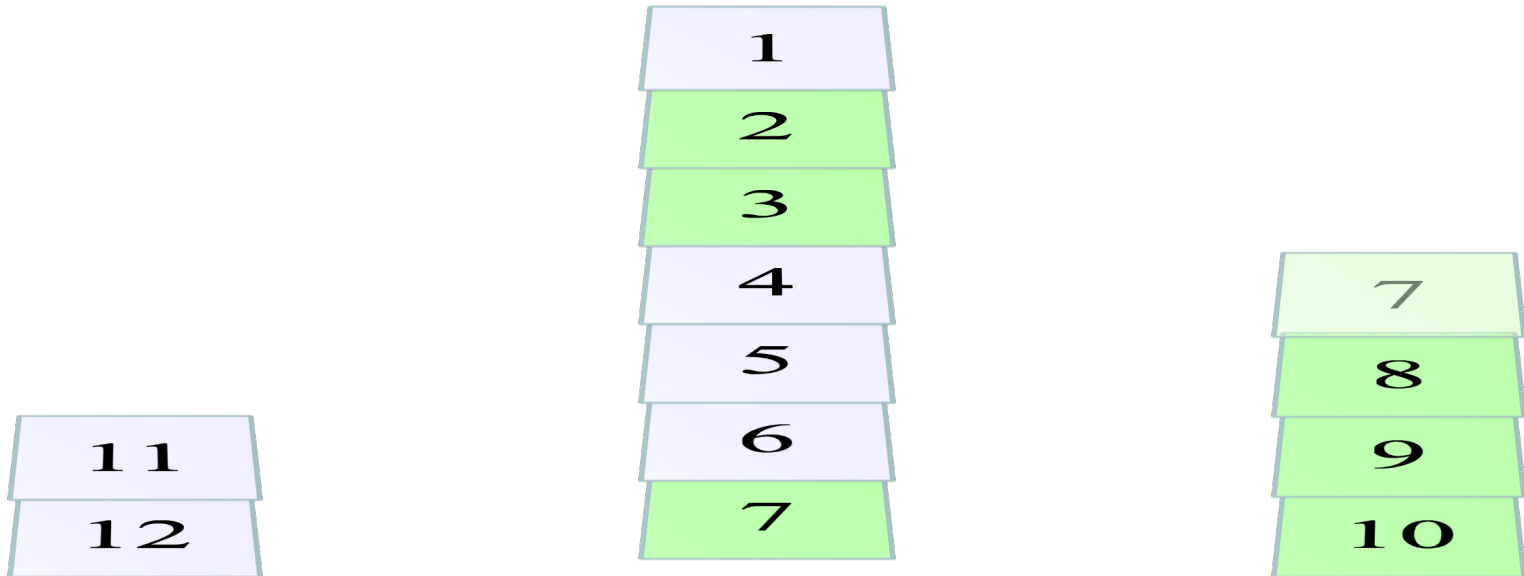
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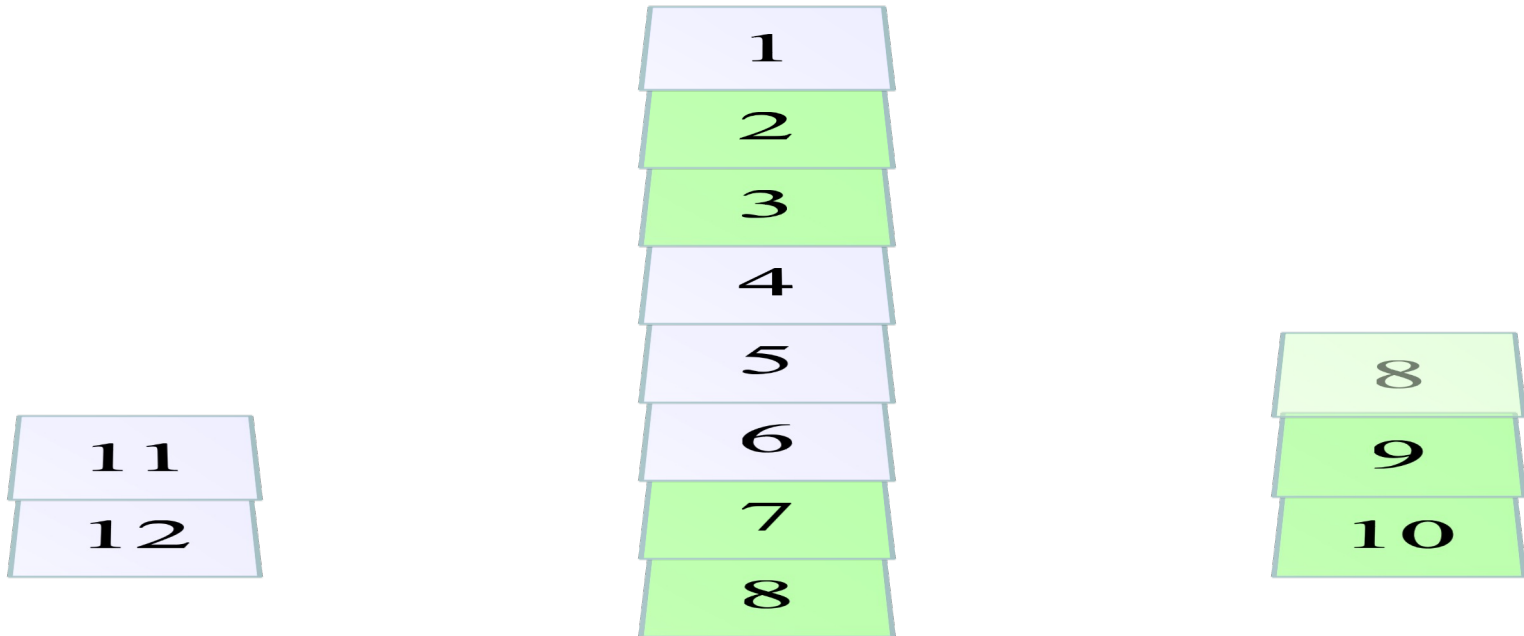
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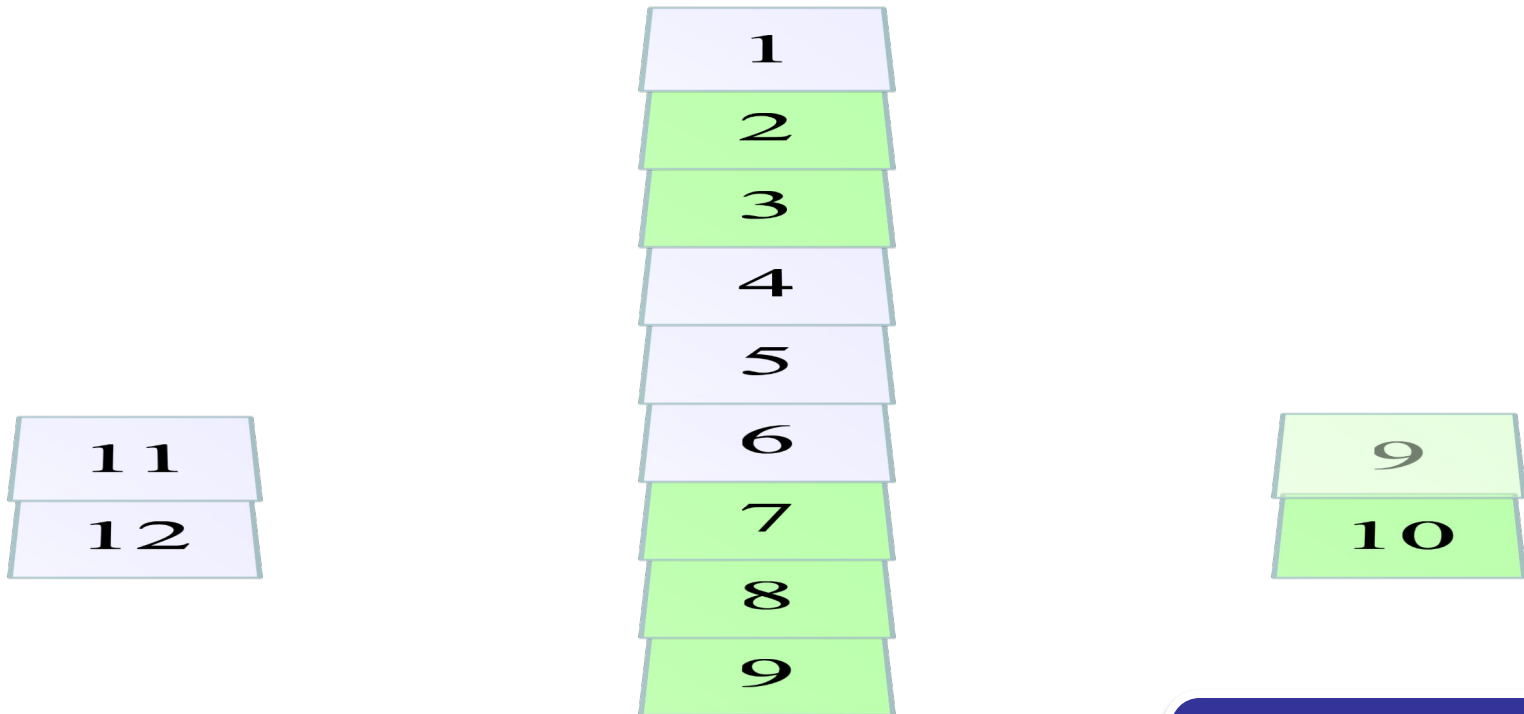
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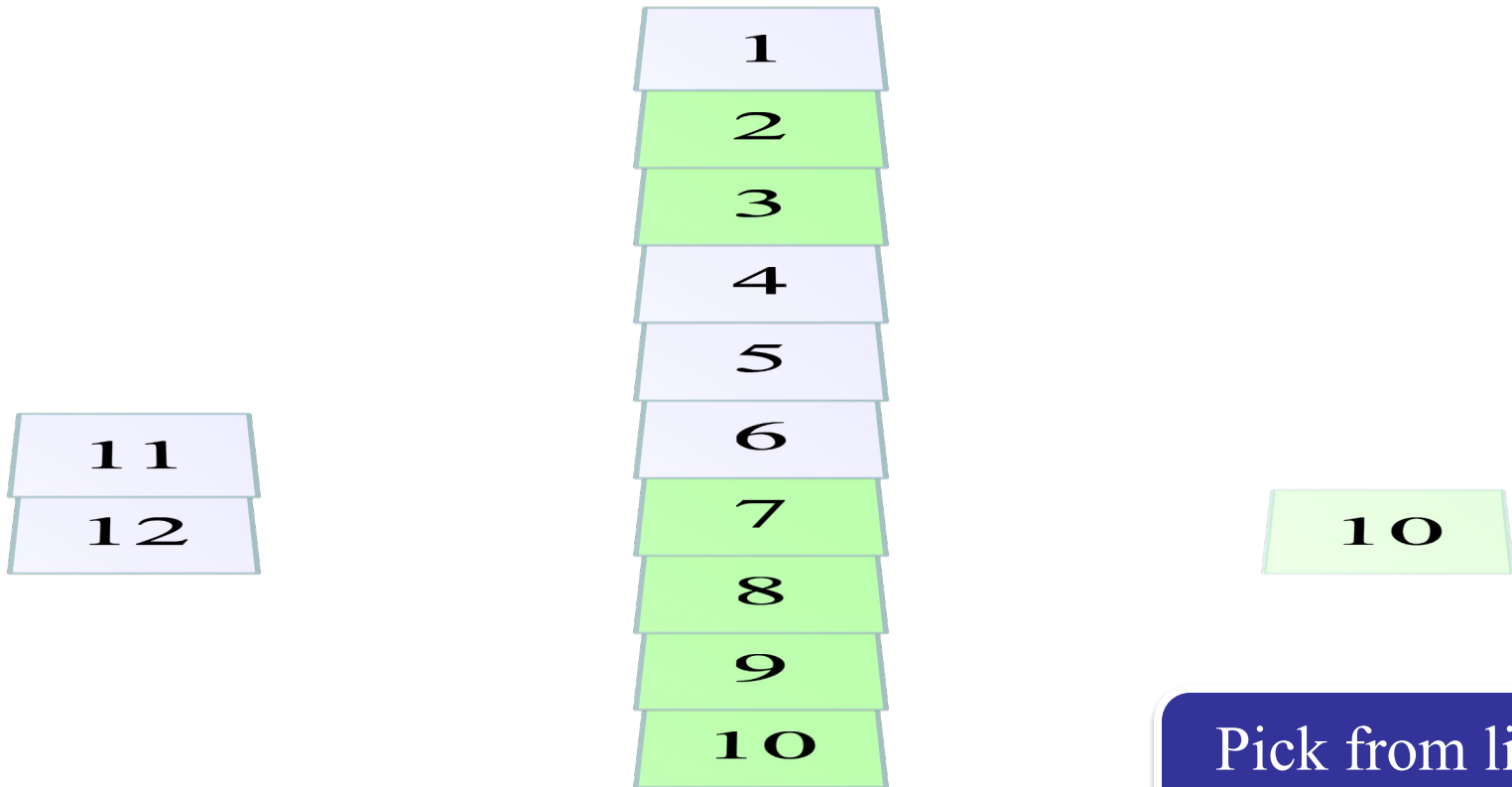
Pick from list
with the least

Combining Two Sorted Lists



Pick from list
with the least

Combining Two Sorted Lists



Combining Two Sorted Lists



Finish off
remaining list

Combining Two Sorted Lists



1 2

Finish off
remaining list

Merge Sort

```
def merge_sort(b, h, k):  
    """Sort the array fragment b[h..k]"""  
    if b[h..k] has fewer than 2 elements:  
        return  
    # Divide and recurse  
    mid = (h+k)//2  
    merge_sort(b, h, m)  
    merge_sort(b, m+1, k)  
    # Combine  
    merge(b,h,mid,k) # Merge halves into b
```

- Seems simpler than **qsort**
 - Straight-forward d&c
 - Merge easy to implement
- What is the **catch**?
 - Merge requires a **copy**
 - We did not allow copies
 - Copying takes n steps
 - But so does merge/partition
- $n \log n$ **ALWAYS**

Merge Sort

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Proof beyond
scope of course

What Does Python Use?

- The `sort()` method is **Timsort**
 - Invented by Tim Peters in 2002
 - Combination of insertion sort and merge sort
- Why a combination of the two?
 - Merge sort requires copies of the data
 - Copying pays off for large lists, but not small lists
 - Insertion sort is not that slow on small lists
 - Balancing two properly still gives $n \log n$

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Quicksort is 1959!

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Most of time
spent here

What Does Python Use?

- The `sort()` method is **Timsort**
 - Invented by Tim Peters in 2002
 - Combination of Merge sort and Insertion sort
- Why a combination?
 - Merge sort is $O(n \log n)$
 - Copying small lists is fast
 - Insertion sort is not that slow on small lists
 - Balancing two properly still gives $n \log n$

This strategy allows
AI to find even **better**
sorting algorithms