Lecture 25

Searching & Sorting

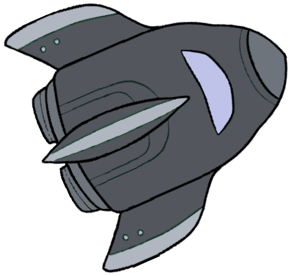
Announcements for This Lecture

Prelim 2

- **Prelim, Tonight at 7:30**
  - **A–L** in Bailey 101
  - **M-Z** in Uris G01
- **Material up to Nov. 8**
  - Recursion + Loops + Classes
- **Graded this Weekend**
  - Grades posted on Monday
  - Need time for make-ups

Assignments

- **A6** still not graded
  - Will be done by after break
  - Staff needs to take their time
- **A7** is due **Monday Dec. 5**
  - Extensions are possible
  - Contact your lab instructor



Linear Search

```
def linear_search(v,b):
    """Returns: first occurrence of v in b (-1 if not found)
    Precond: b a list of number, v a number
    """
    # Loop variable
    i = 0
    while i < len(b) and b[i] != v:
        i = i + 1

    if i == len(b):  # not found
        return -1
    return i
```

How many entries do we have to look at?

Linear Search

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def linear_search(v,b):
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```

How many entries do we have to look at?

All of them!

Linear Search

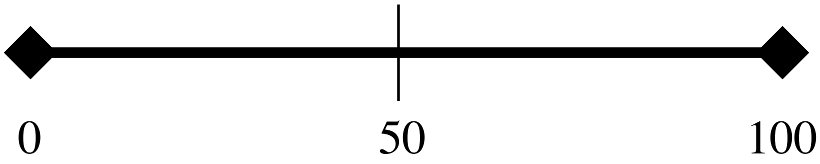
```
def linear_search(v,b):
    """Returns: last occurrence of v in b (-1 if not found)
    Precond: b a list of number, v a number
    """
    # Loop variable
    i = len(b)-1
    while i >= 0 and b[i] != v:
        i = i - 1

    # Equals -1 if not found
    return i
```

How many entries do we have to look at?

All of them!

Is There a Better Way?



- Thinking of number 0..100
  - You get to guess number
  - I tell you higher or lower
  - Continue until get it right
- **Goal:** Keep # guesses low
  - Use my answers to help
- **Strategy?**
  - Start guess in the middle
  - Answer eliminates half
  - Go to middle of remaining

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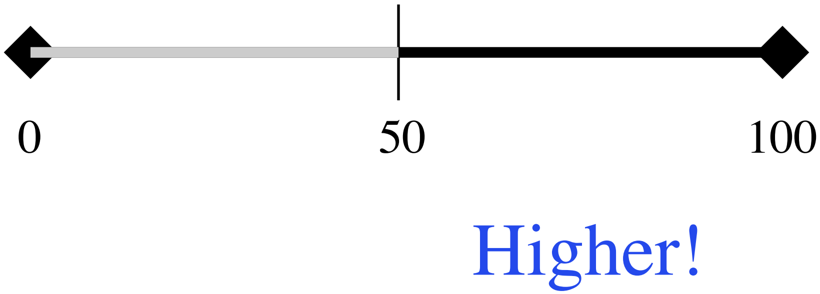

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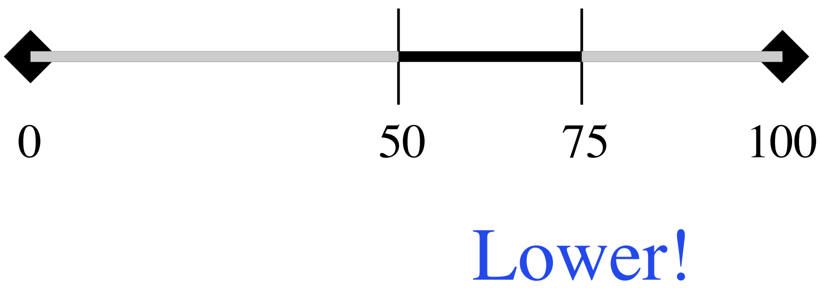

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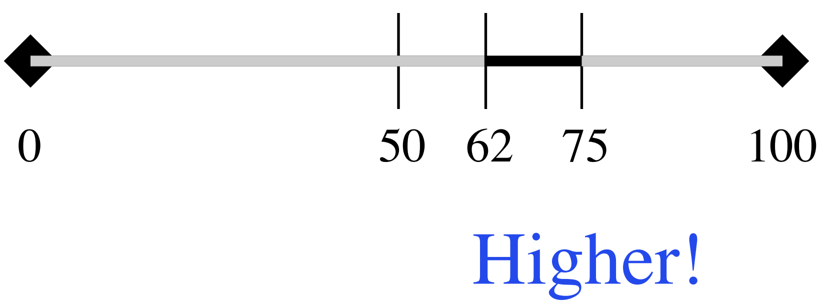

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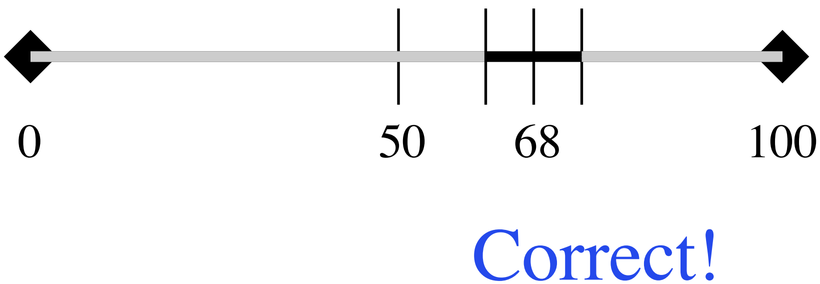

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- **Goal:** Keep # guesses low
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Binary Search

```
def binary_search(v,b):
    # Loop variable(s)
    i = 0, j = len(b)
    while i < j and b[i] != v:
        mid = (i+j)//2
        if b[mid] < v:
            i = mid+1
        elif b[mid] > v:
            j = mid
        else:
            return mid
    return -1 # not found
```

Requires that the data is sorted!

But few checks!

Observation About Sorting

- Sorting data can speed up searching
  - Sorting takes time, but do it once
  - Afterwards, can search many times
- Not just searching. Also speeds up
  - Duplicate elimination in data sets
  - Data compression
  - Physics computations in computer games
- Why it is a major area of computer science

The Sorting Challenge

- **Given:** A list of numbers
- **Goal:** Sort those numbers using only
  - Iteration (while-loops or for-loops)
  - Comparisons (< or >)
  - Assignment statements
- Why? For proper **analysis**.
  - Methods/functions come with hidden costs
  - Everything above has no hidden costs
  - Each comparison or assignment is “1 step”

This Requires Some Notation

- As the list is sorted…
  - Part of the list **will** be sorted
  - Part of the list will **not** be sorted
- Need a way to refer to portions of the list
  - Notation to refer to sorted/unsorted parts
- And have to do it **without** slicing!
  - Slicing makes a **copy**
  - Want to sort original list, not a copy

This Requires Some Notation

- As the list is sorted…
  - Part of the list **will** be sorted
  - Part of the list will **not** be sorted
- Need a w… list
  - Notatio…

But we will be less formal than in previous years!

- And have to do it **without** slicing!
  - Slicing makes a **copy**
  - Want to sort original list, not a copy

Recall: Range Notation

- m..n is a range containing n+1-m values
  - 2..5 contains 2, 3, 4, 5. Contains 5+1 – 2 = 4 values
  - 2..4 contains 2, 3, 4. Contains 4+1 – 2 = 3 values
  - 2..3 contains 2, 3. Contains 3+1 – 2 = 2 values
  - 2..2 contains 2. Contains 2+1 – 2 = 1 values
  - 2..1 contains ???
- The notation m..n, always implies that m <= n+1
  - So you can assume that even if we do not say it
  - If m = n+1, the range has 0 values

Recall: Range Notation

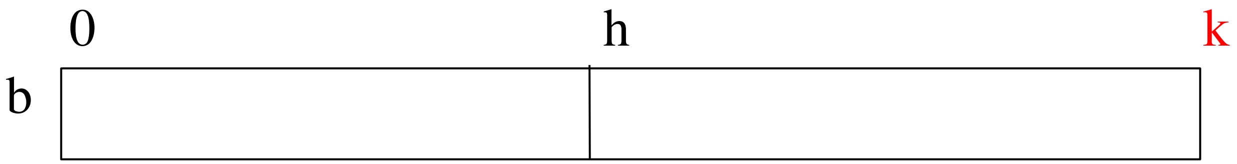
- m..n is a range containing n+1-m values
  - 2..5 contains 2, 3, 4, 5. Co … lues
  - 2..4 contains 2, 3, 4. Co … ues
  - 2..3 contains 2, 3. Co … ues
  - 2..2 contains 2. Contains 2+1 – 2 = 1 values
  - 2..1 contains ???

Not the same as range(m,n)

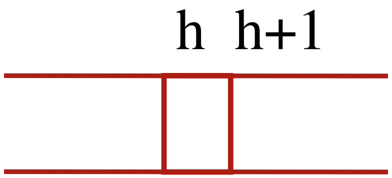
- The notation m..n, always implies that m <= n+1
  - So you can assume that even if we do not say it
  - If m = n+1, the range has 0 values

Horizontal Notation

- Want a pictoral way to visualize this sorting
  - Represent the list as long rectangle
  - We saw this idea in divide-and-conquer



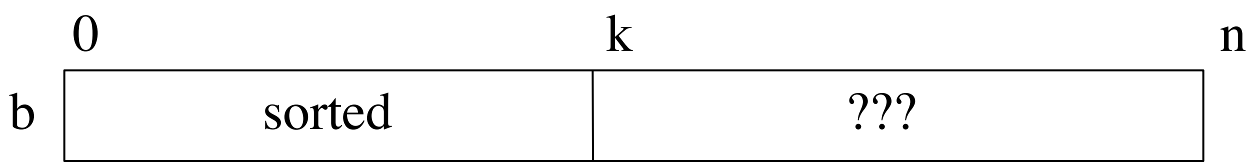
- Do **not** show individual boxes
  - Just dividing lines between regions
  - Label dividing lines with indices
  - But index is either left or right of dividing line



(h+1) – h = 1

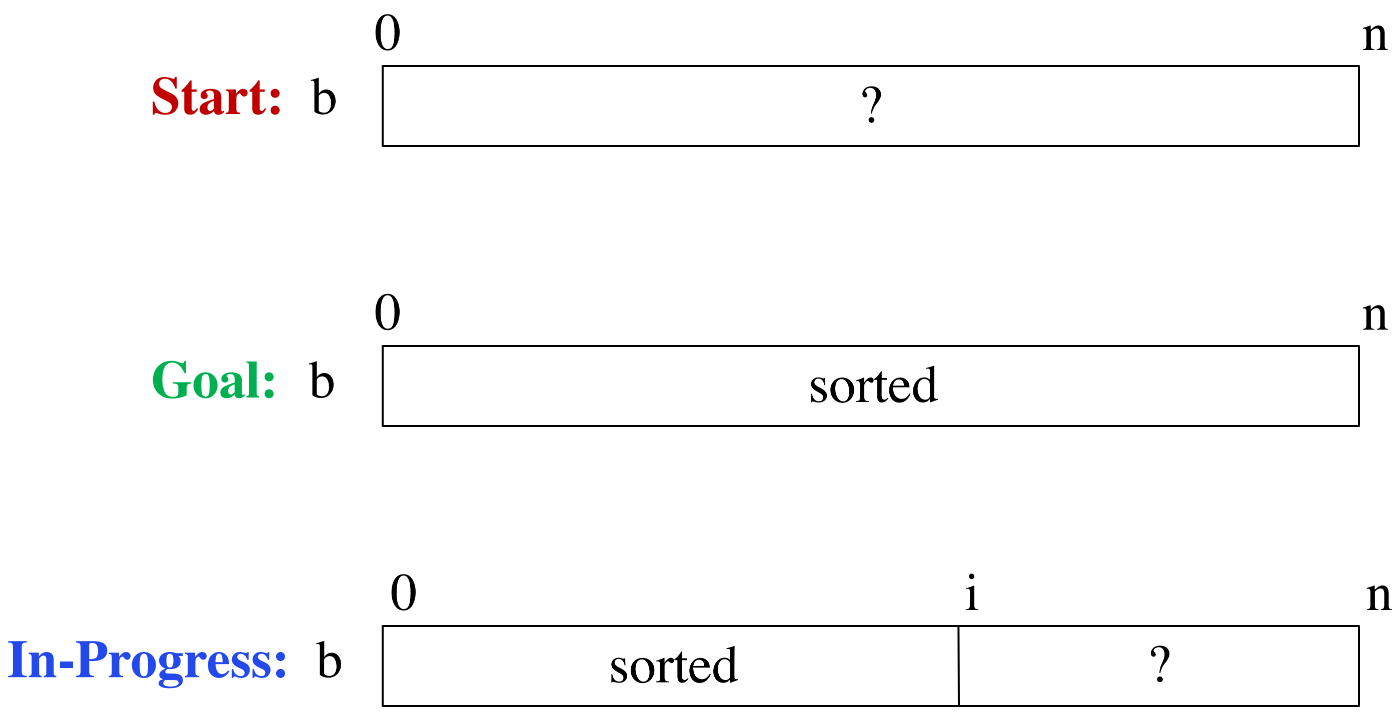
Horizontal Notation

- Label regions with properties
  - **Example:** Sorted or ???



  - `b[0..k-1]` is sorted
  - `b[k..n-1]` **unknown** (might be sorted)
- Picture allows us to track progress

Visualizing Sorting



Insertion Sort

	0	i	n
b	sorted	?	

```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

0						i
2	4	4	6	6	7	5

0						i
2	4	4	5	6	6	7

Remember the restrictions!

Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

swap shown in the lecture about lists

0						i	
2	4	4	6	6	7	5	

Insertion Sort: Moving into Position

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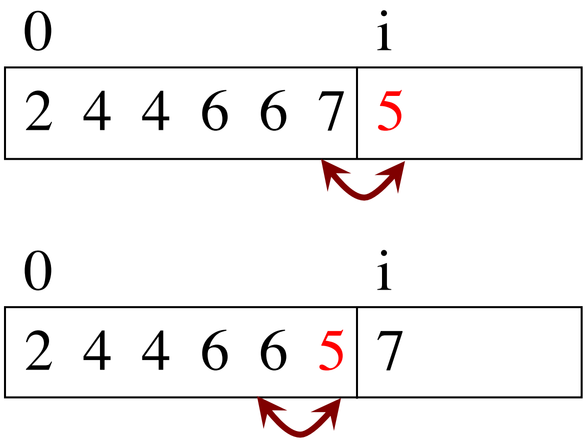

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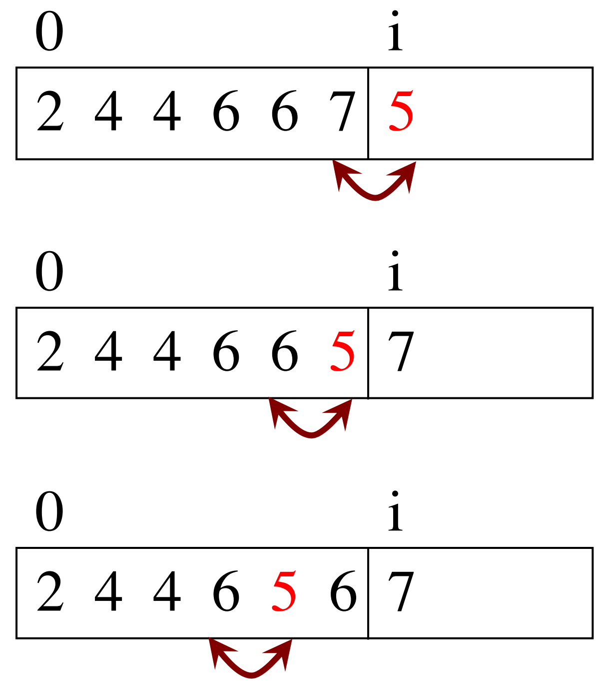

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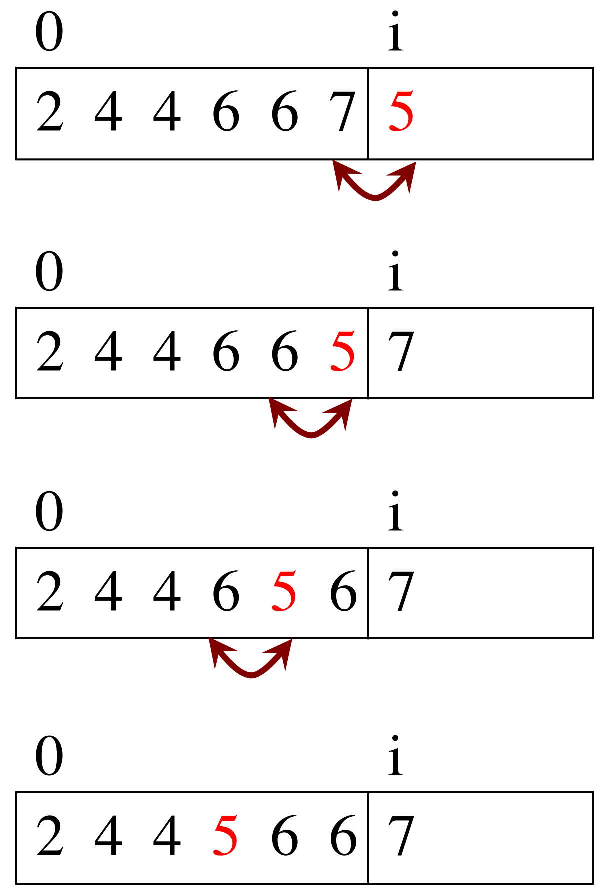

The Importance of Helper Functions

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

VS

Can you understand all this code below?

```
i = 0
while i < n:
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            temp = b[j]
            b[j] = b[j-1]
            b[j-1] = temp
        j = j -1
    i = i +1
```

Measuring Performance

- Performance is a tricky thing to measure
  - Different computers run at different speeds
  - Memory also has a major effect as well
- Need an independent way to measure
  - Measure in terms of “basic steps”
  - **Example:** Searching counted # of checks
- For sorting, we measure in terms of **swaps**
  - Three assignment statements
  - Present in all sorting algorithms

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

Total Swaps: $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2 = (n^2-n)/2$

Insertion Sort: Performance

```
def push_down(b, i):
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    j = i
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Insertion sort is an n^2 algorithm

Total Swaps: $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2 = (n^2-n)/2$

Algorithm "Complexity"

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
log n	0.003 s	0.006 s	0.01 s
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

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Binary Search

Linear Search

Insertion Sort

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- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
log n	[illegible]	[illegible]	0.01 s
n	[illegible]	[illegible]	1 s
n log n	[illegible]	[illegible]	4.79 s
n^2	[illegible]	[illegible]	16.7 m
n^3	1 s	16.7 m	11.6 d
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Major Topic in 2110:
Beyond scope of this course

Insertion Sort is Not Great

- Typically n^2 is okay, but not great
  - Will perform horribly on large data
  - Very bad when performance critical (games)
- We would like to do better than this
  - Can we get n swaps (**no**)?
  - How about n log n (**maybe**)
- This will require a new algorithm
  - Let's return to horizontal notation

A New Algorthm

	0		n
Start: b	?		

	0		n
Goal: b	sorted		

	0	i	n
In-Progress: b	sorted, ≤ b[i..]	≥ b[0..i-1]	

First segment always contains smaller values

Selection Sort

	0	i	n
b	sorted , ≤ b[i..]	≥ b[0..i-1]	

```
i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
```

					i						n
2	4	4	6	6	8	9	9	7	8	9	

					i						n
2	4	4	6	6	7	9	9	8	8	9	

						i					n
2	4	4	6	6	7	9	9	8	8	9	

Remember the restrictions!

Selection Sort

How fast is this?

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

	i										n
2	4	4	6	6	8	9	9	7	8	9	

	i										n
2	4	4	6	6	7	9	9	8	8	9	

	i										n
2	4	4	6	6	7	9	9	8	8	9	

Selection Sort

This is also n^2!

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

This is n steps

					i						n
2	4	4	6	6	8	9	9	7	8	9	

					i						n
2	4	4	6	6	7	9	9	8	8	9	

						i					n
2	4	4	6	6	7	9	9	8	8	9	

What is the Problem?

- Both insertion, selection sort are **nested loops**
  - **Outer loop** over each element to sort
  - **Inner loop** to put next element in place
  - Each loop is n steps. $n \times n = n^2$
- To do better we must *eliminate* a loop
  - But how do we do that?
  - What is like a loop? **Recursion!**
  - Will see how to do this next lecture