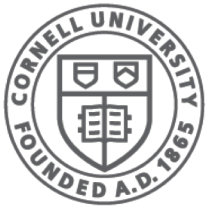
http://www.cs.cornell.edu/courses/cs1110/2019sp

# Lecture 16:
# More Recursion!

## CS 1110

## Introduction to Computing Using Python




Cornell CIS
COMPUTING AND INFORMATION SCIENCE

[E. Andersen, A. Bracy, D. Gries, L. Lee, S. Marschner, C. Van Loan, W. White]

# Recursion

**Recursive Function**:

A function that calls itself (directly or indirectly)

**Recursive Definition**:

A definition that is defined in terms of itself

# A Mathematical Example: Factorial

**Non-recursive definition:**

$$n! = n \times n\text{-}1 \times \ldots \times 2 \times 1$$

$$= n\,(n\text{-}1 \times \ldots \times 2 \times 1)$$

**Recursive definition:**

$n! = n\,(n\text{-}1)!$ for $n > 0$ **Recursive case**

$0! = 1$ **Base case**

What happens if there is no base case?

# Recursive Call Frames

```
def factorial(n):
    """Returns: factorial of n.
    Precondition: n ≥ 0 an int"""
1   if n == 0:
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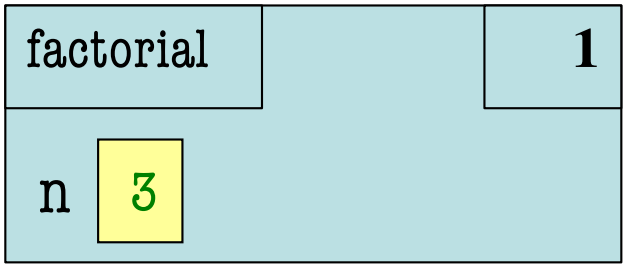
3   return n*factorial(n-1)


factorial(3)
```



# Recursive Call Frames

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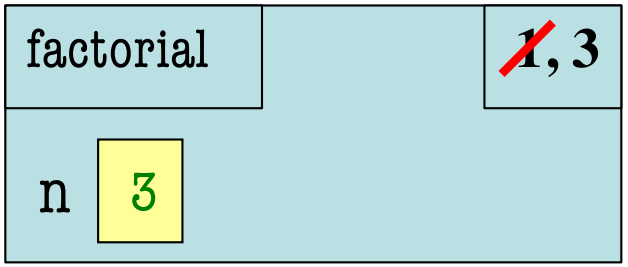


**Call:** factorial(3)

# Recursion

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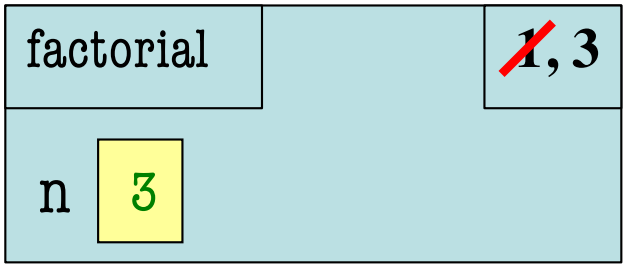


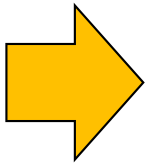

Now what?
Each call is a new frame.

factorial(3)

# What happens next? (Q)

```
def factorial(n):
    """Returns: factorial of n.
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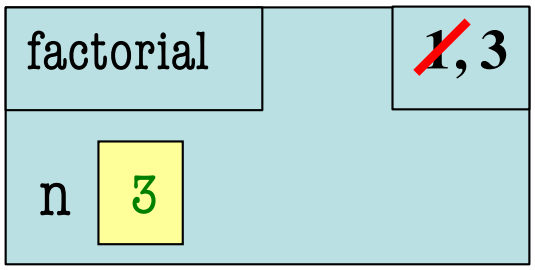
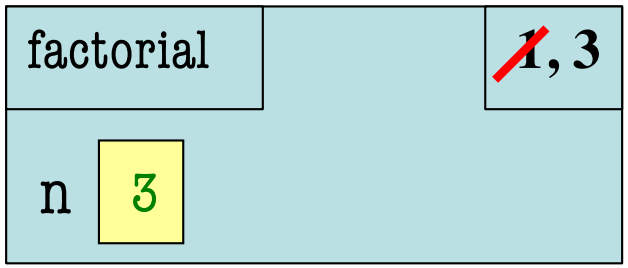
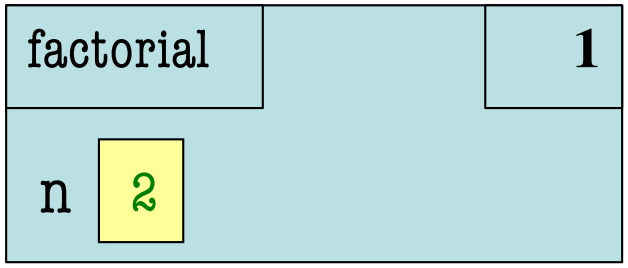
```

**Call:** factorial(3)

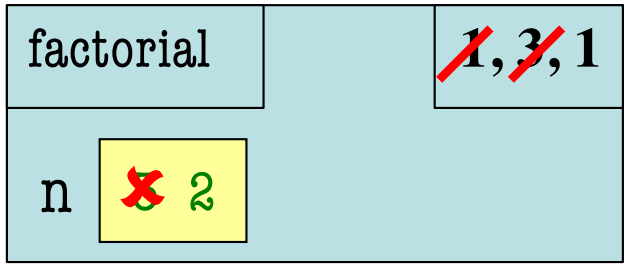



A:

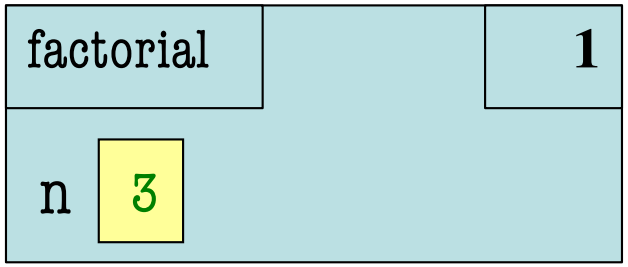


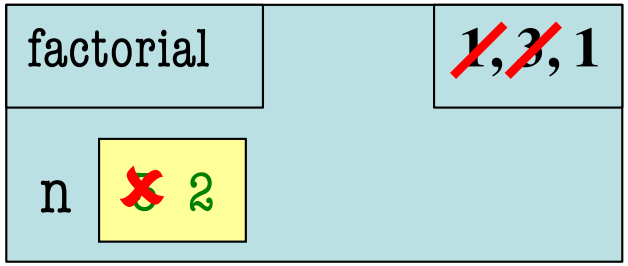

B:

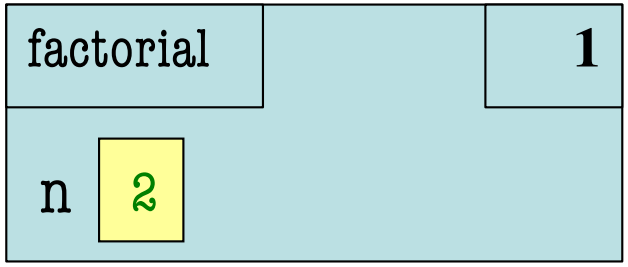



C: **ERASE FRAME**




D:



# What happens next? (A)

```
def factorial(n):
    """Returns: factorial of n.
    Pre: n ≥ 0 an int"""
1   if n == 0:
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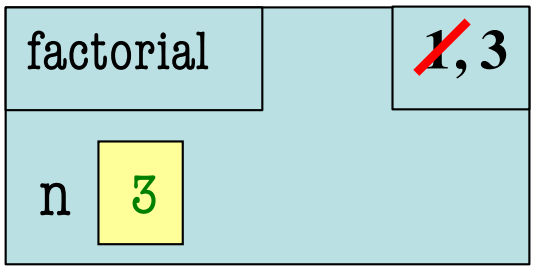
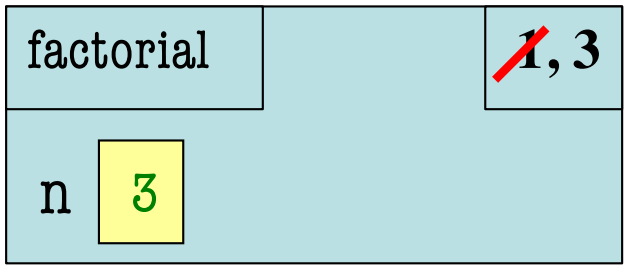
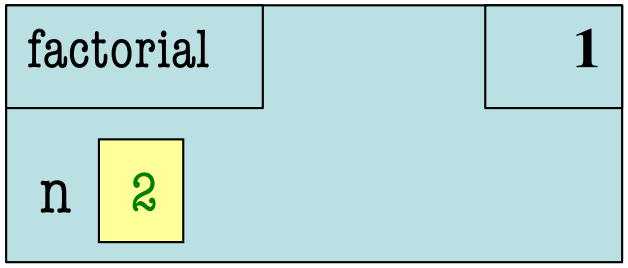
3   return n*factorial(n-1)
```

**Call:** factorial(3)




## A: CORRECT

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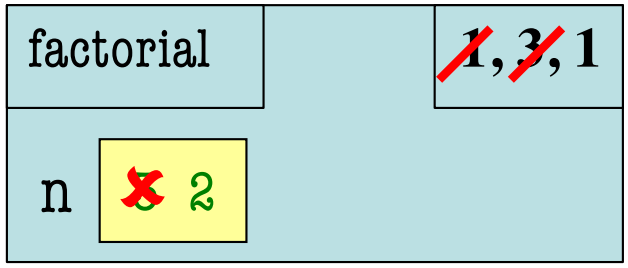
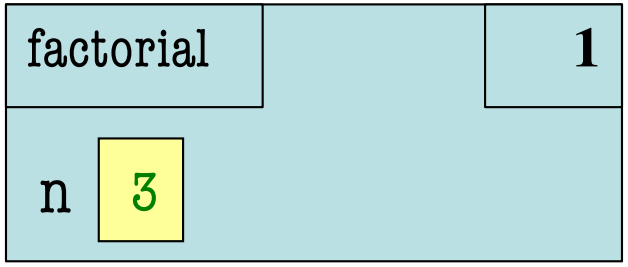



## C: ERASE FRAME

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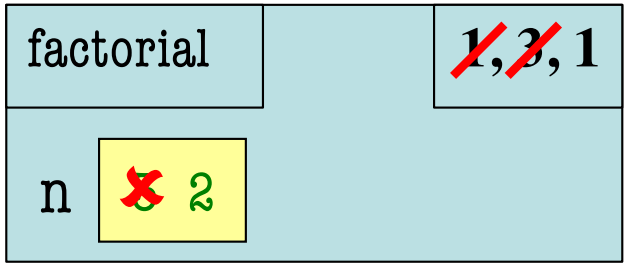


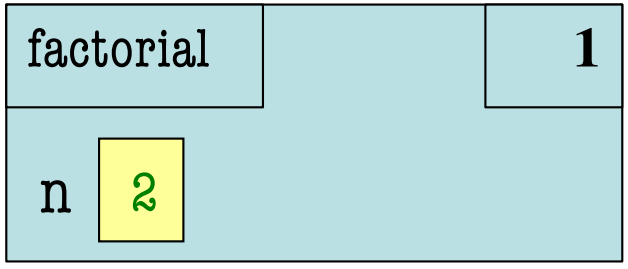

# Recursive Call Frames

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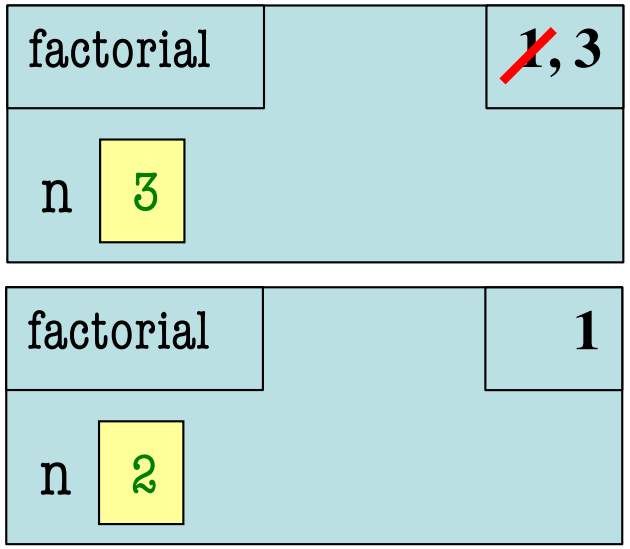


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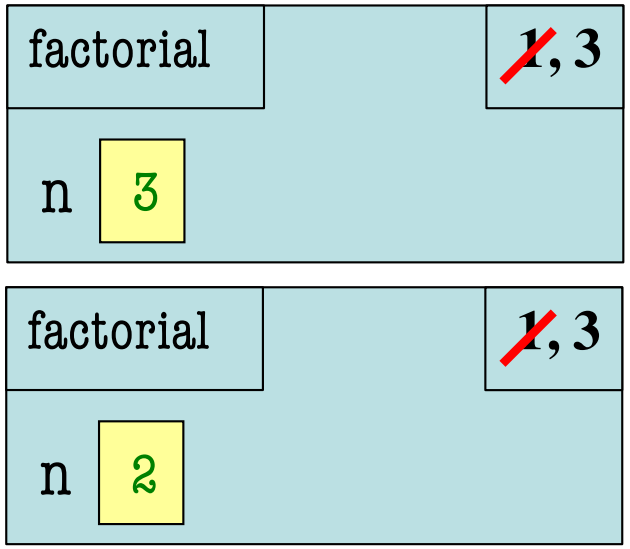


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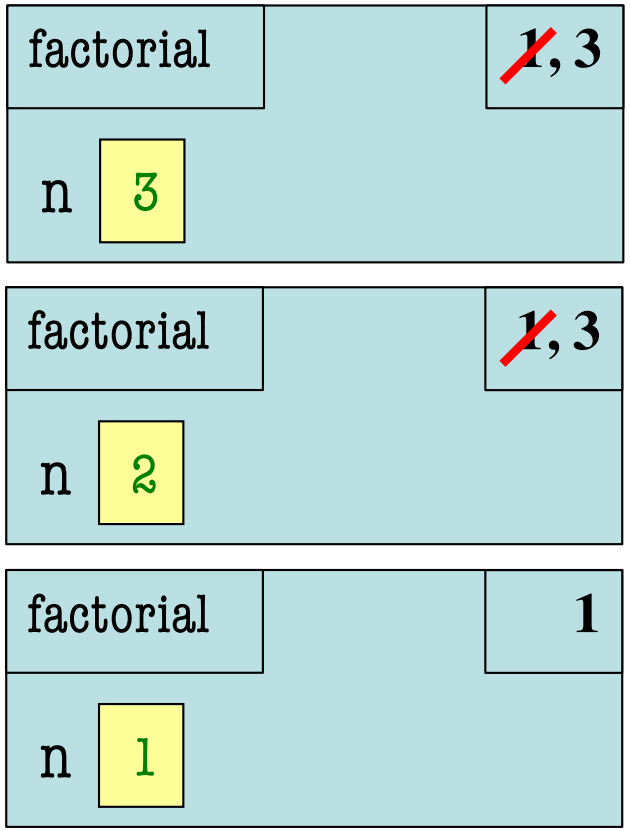


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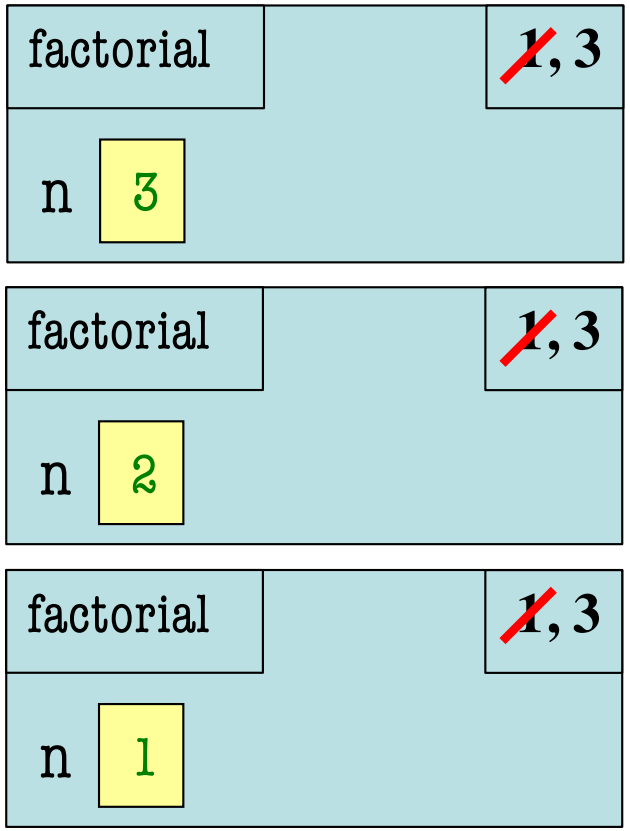


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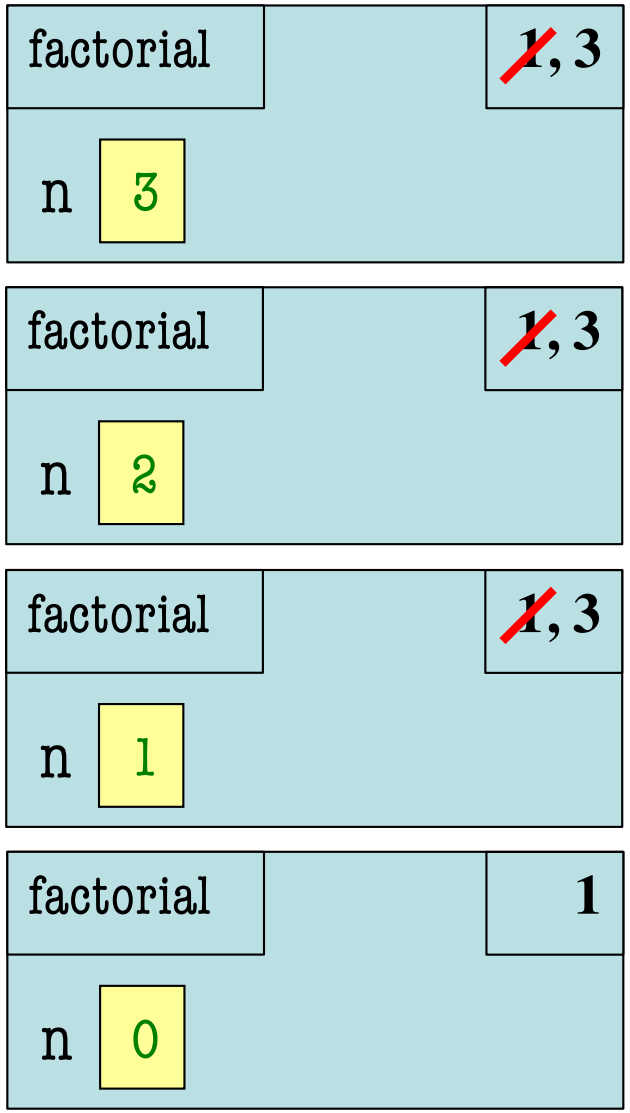

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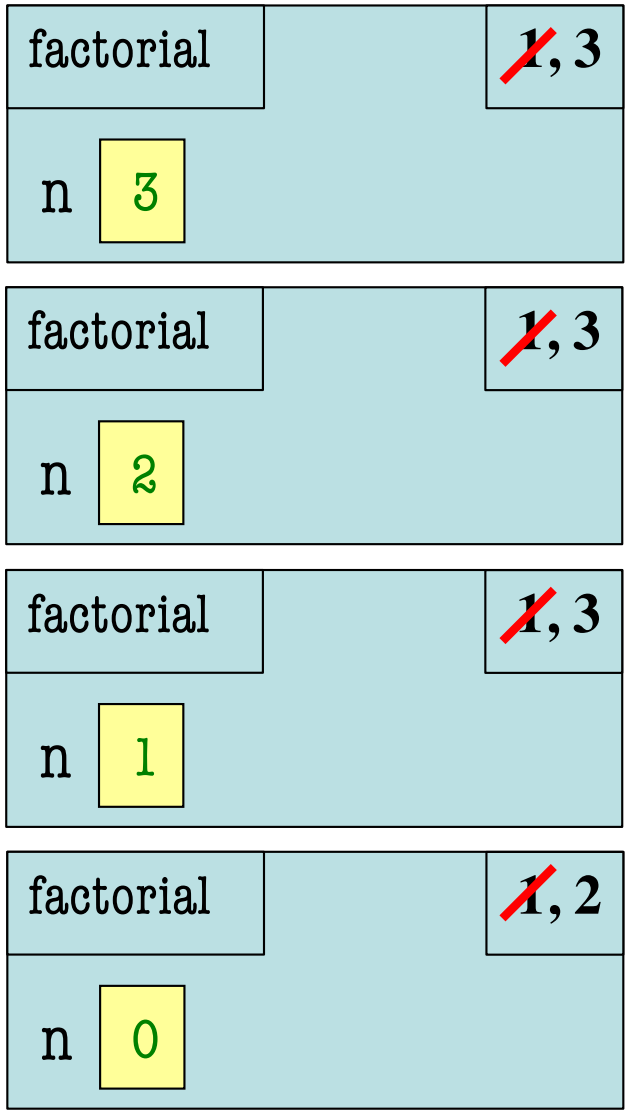

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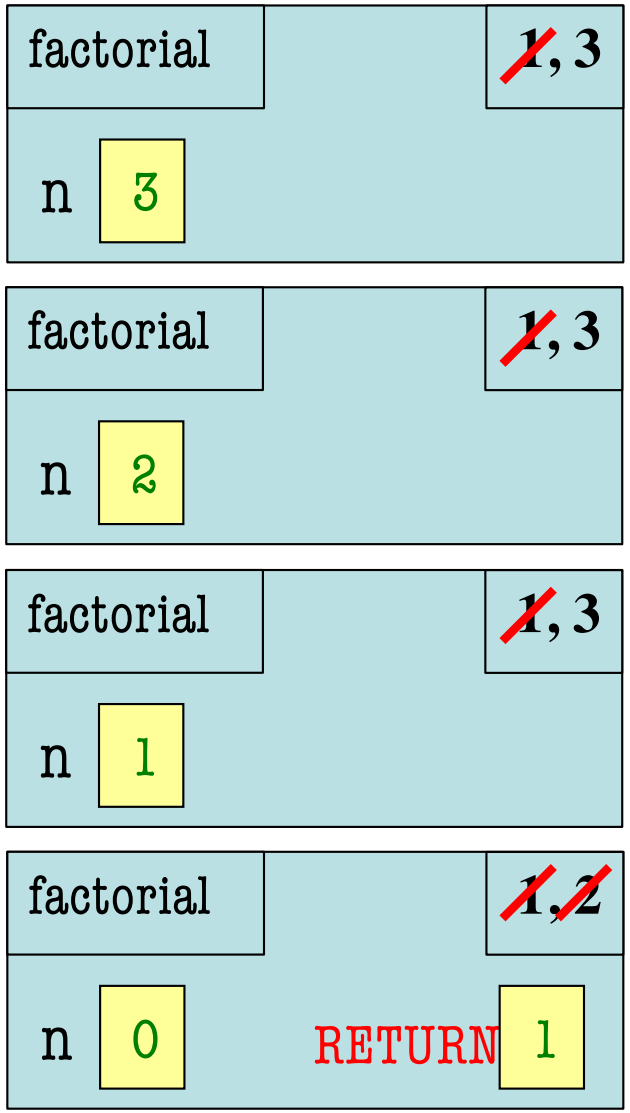

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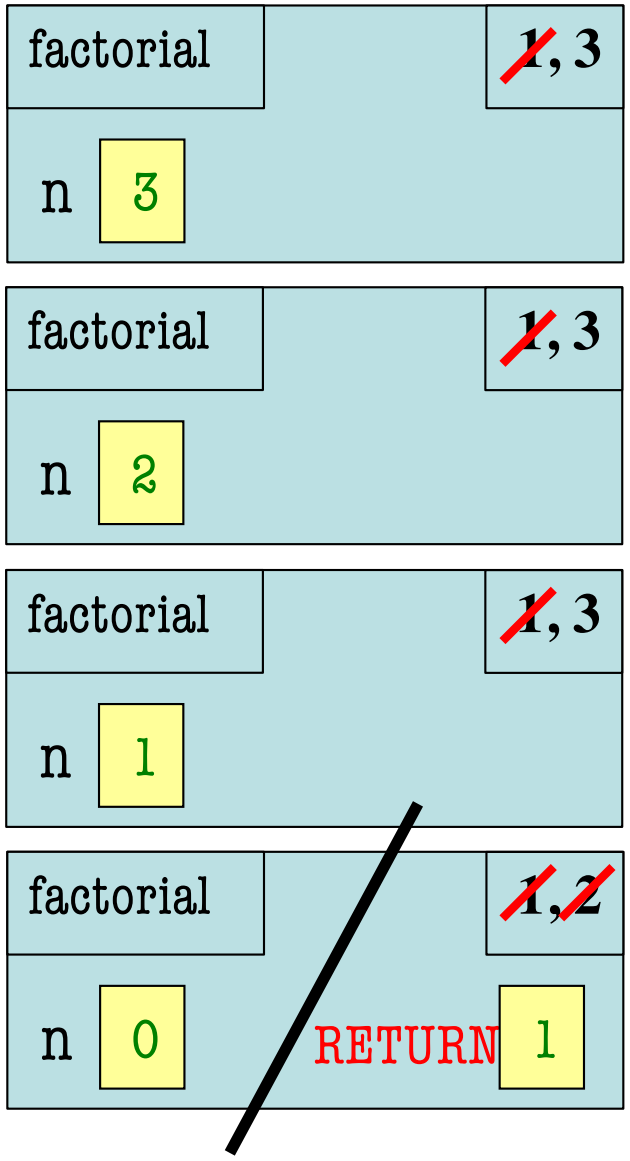

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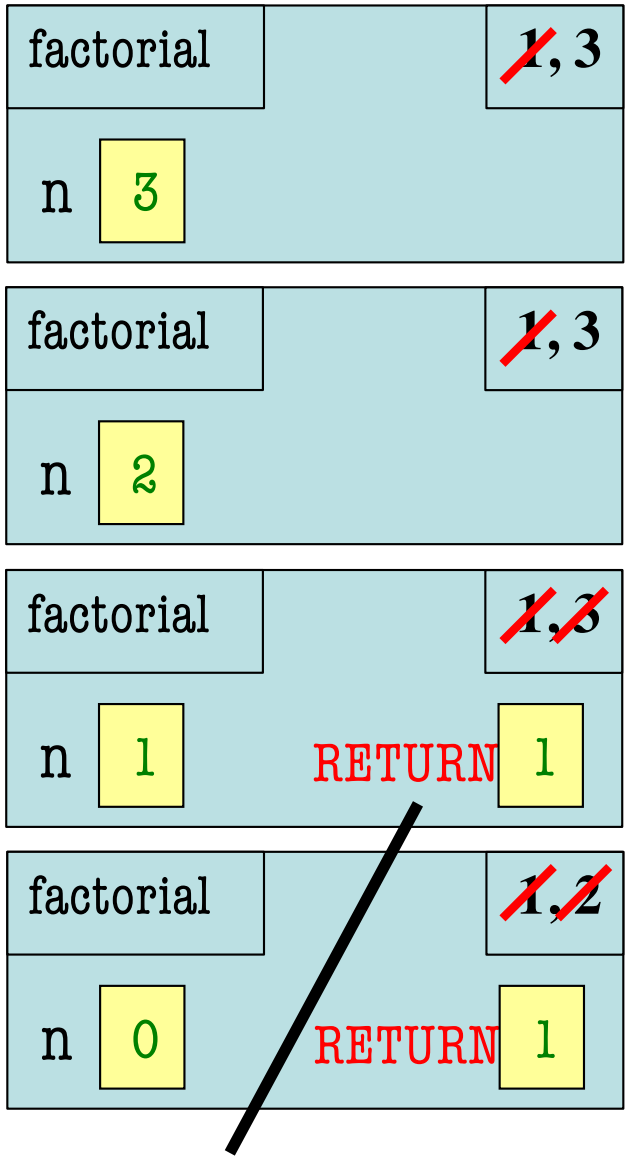

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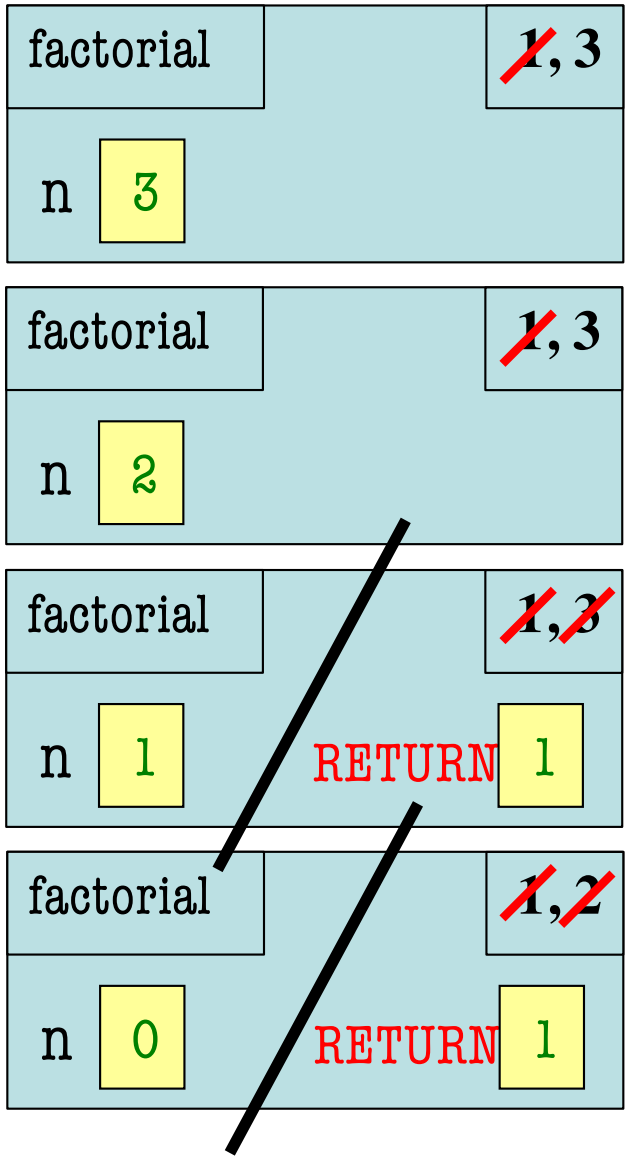

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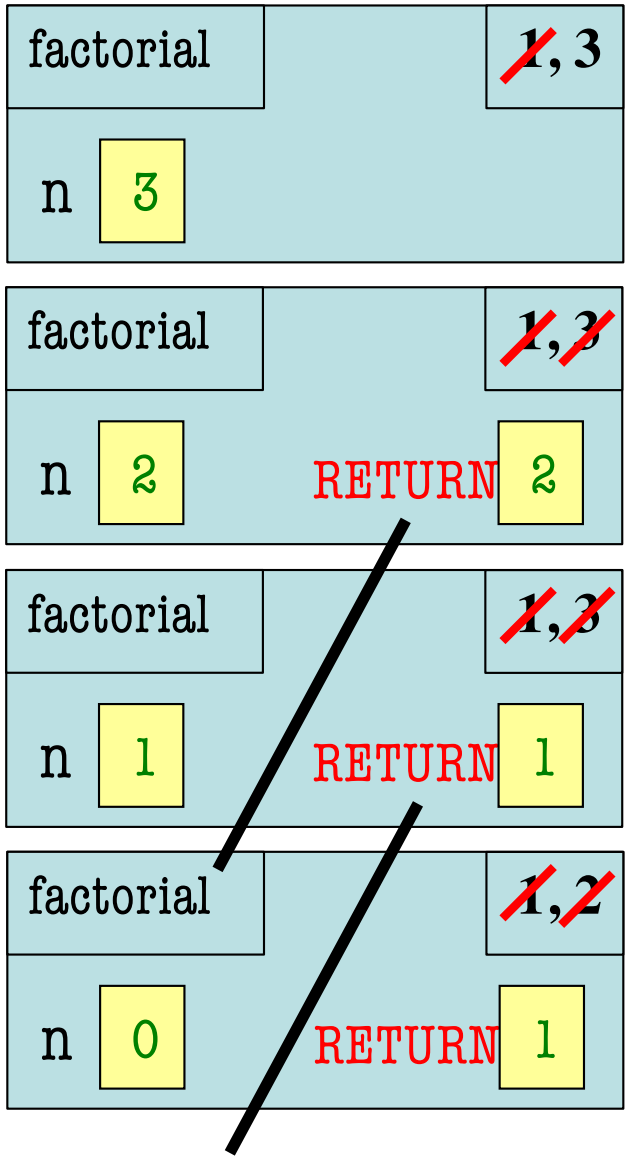

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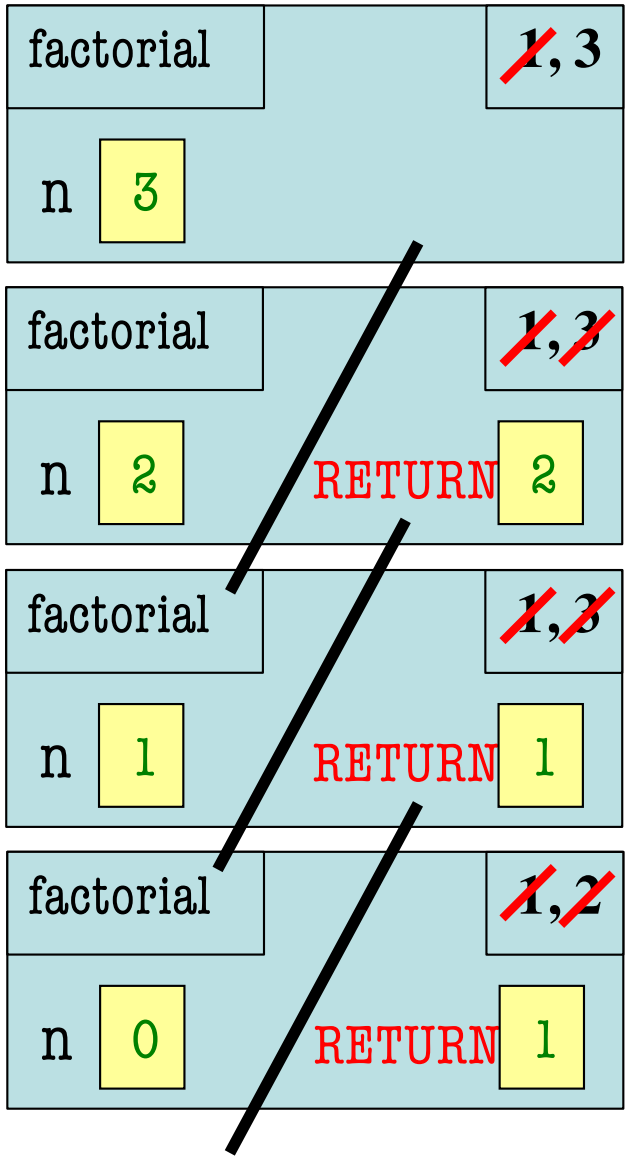

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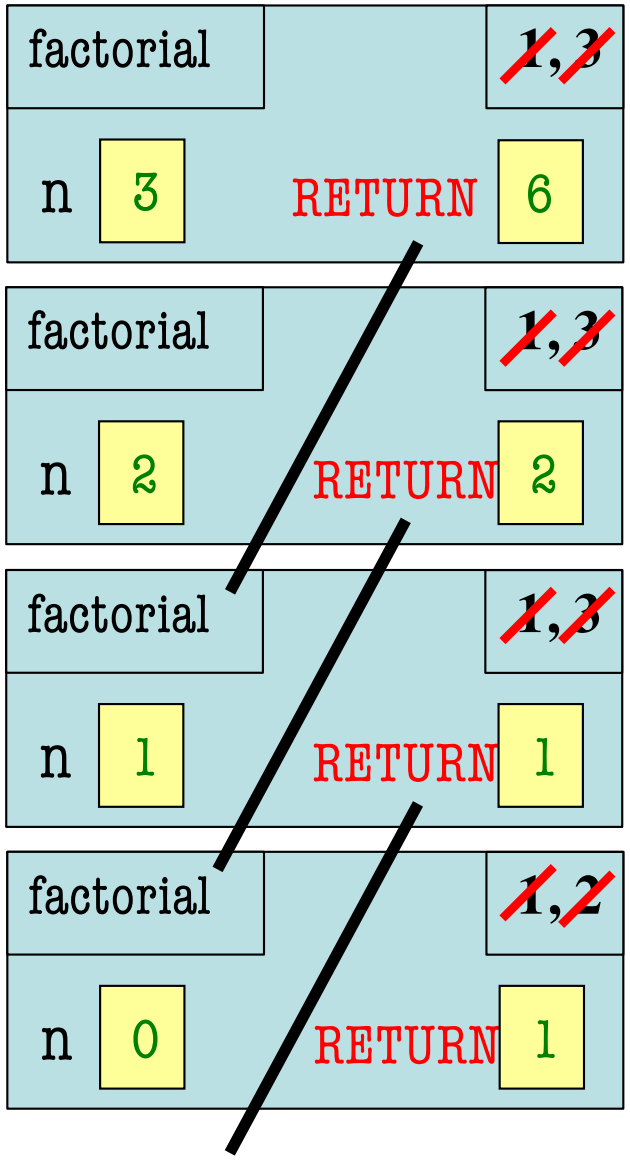

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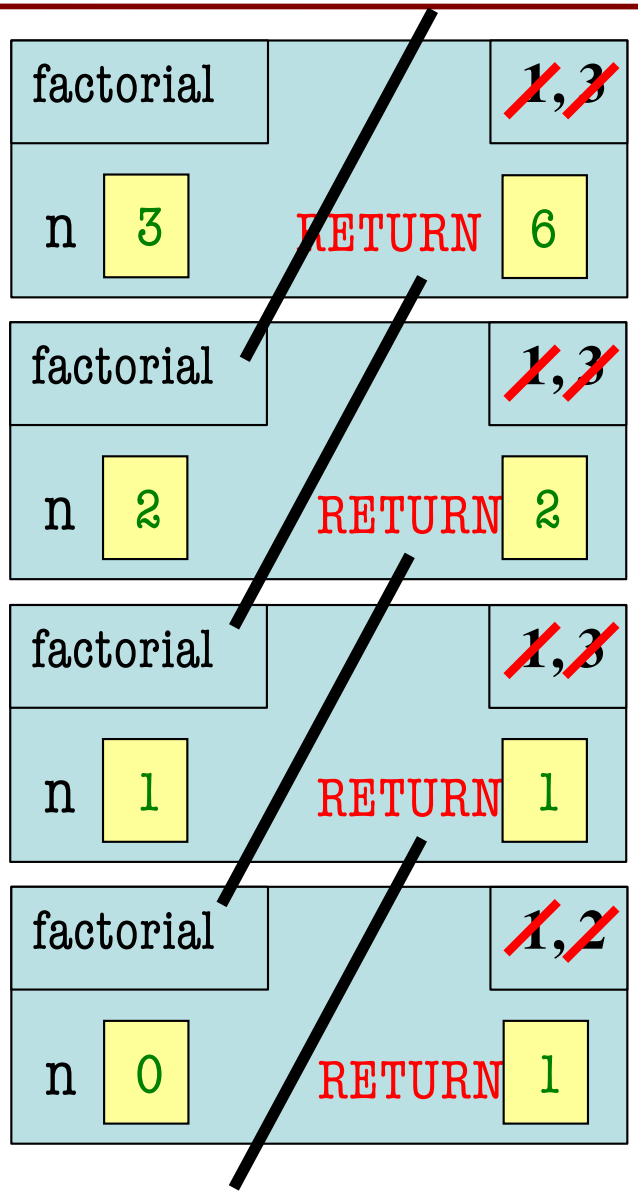
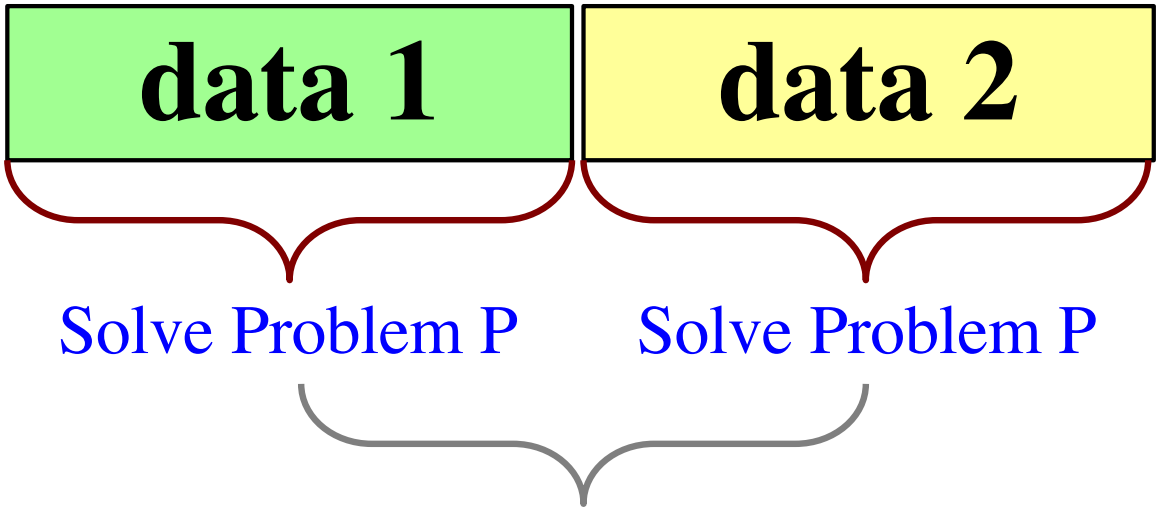


# Divide and Conquer

**Goal**: Solve problem P on a piece of data


**Idea**: Split data into two parts and solve problem



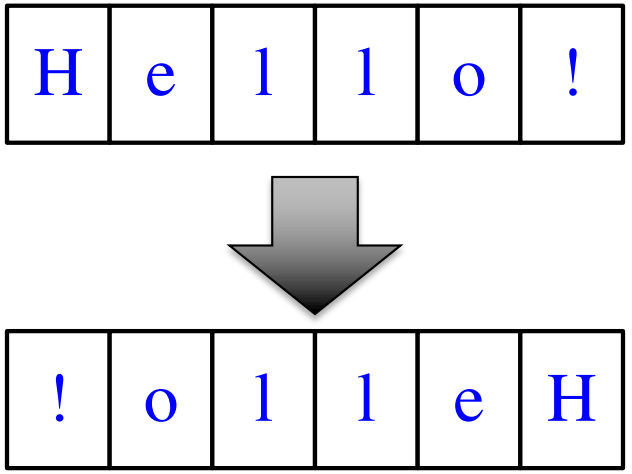
# Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case


    # 2. Break into two parts


    # 3. Combine the result
```



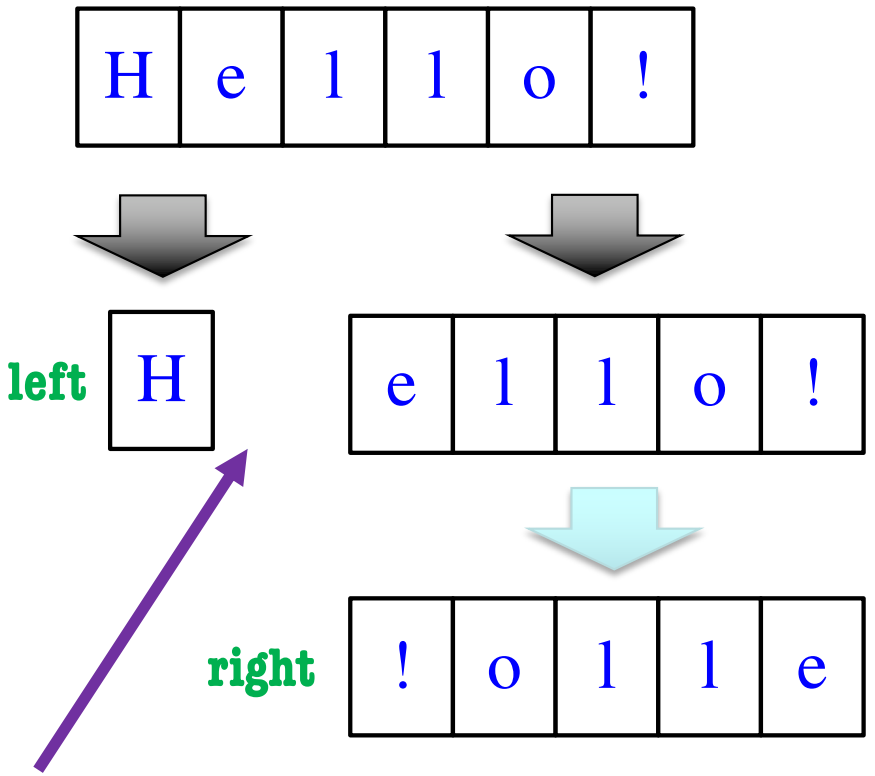
# Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case


    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
```



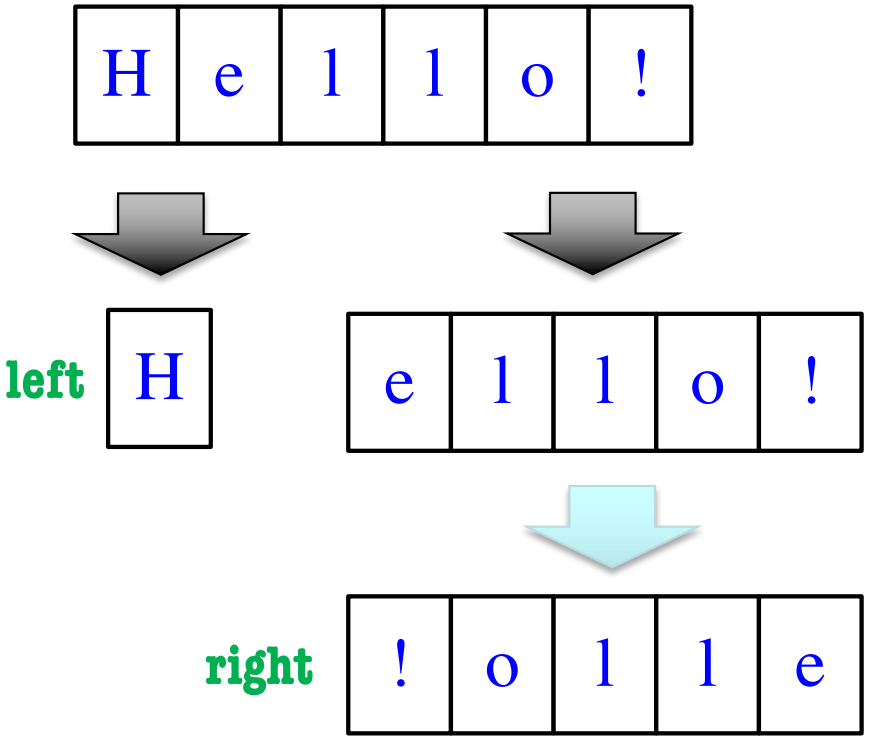

*If this is how we break it up….*

*How do we combine it?*

# How to Combine? (Q)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
```




```
    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
    return
```

A: left + right | B: right + left | C: left | D: right

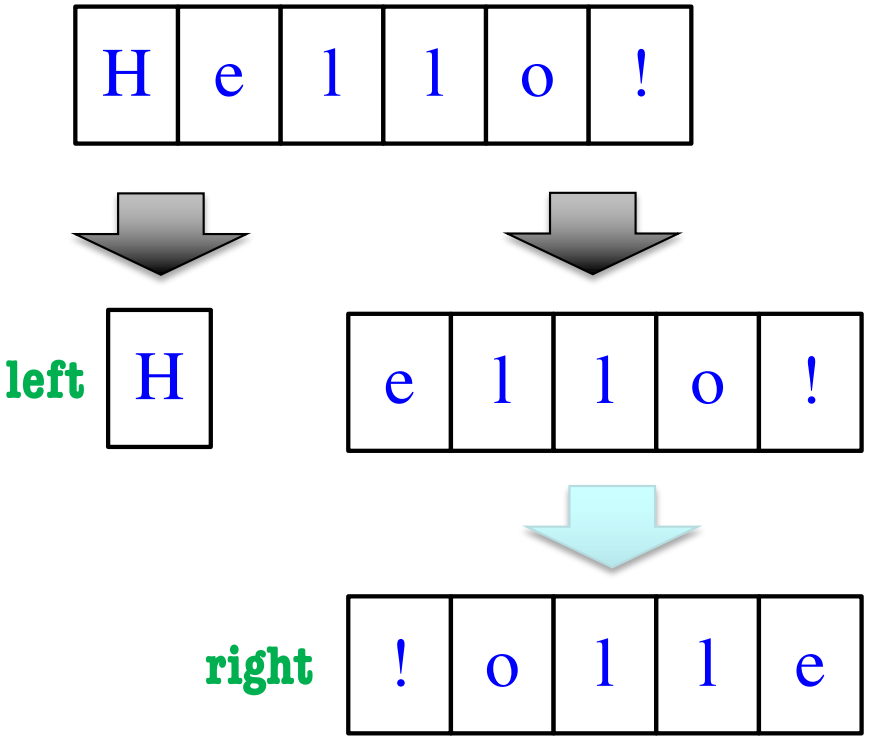
# How to Combine? (A)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case


    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
    return
```




**CORRECT**

| A: left + right | B: right + left | C: left | D: right |
|---|---|---|---|

# Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case




    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
    return right+left
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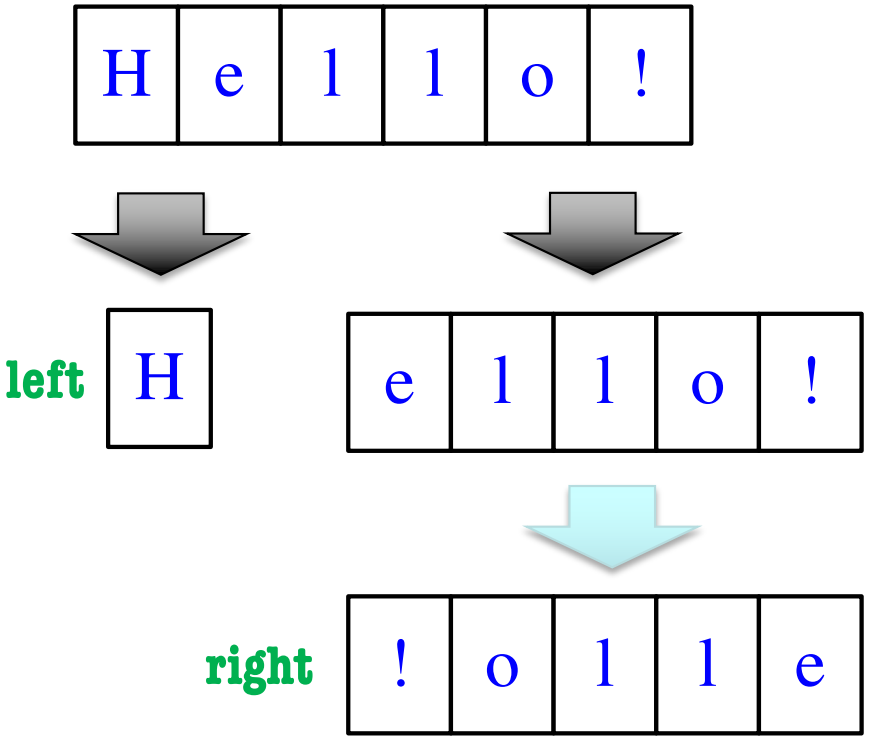


# What is the Base Case? (Q)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
```

| H | e | l | l | o | ! |
|---|---|---|---|---|---|

A: `if s == "":`
`return s`

B: `if len(s) <= 2:`
`return s`

C: `if len(s) <= 1:`
`return s`

```
    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
    return right+left
```

D: Either A or C would work

E: A, B, and C would all work

# What is the Base Case? (A)

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
```

| H | e | l | l | o | ! |
|---|---|---|---|---|---|

**CORRECT**

A:
```
if s == "":
    return s
```

B:
```
if len(s) <= 2:
    return s
```

C:
```
if len(s) <= 1:
    return s
```

```
    # 2. Break into two parts
    left  = reverse(s[0])
    right = reverse(s[1:])

    # 3. Combine the result
    return right+left
```

D: Either A or C would work

E: A, B, and C would all work

# Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle base case
    if len(s) <= 1:
        return s

    # 2. Break into two parts
    left  = reverse(s[0])    s[0]
    right = reverse(s[1:])

    # 3. Combine the result
    return right+left
```

Base Case

Recursive Case

# Alternate Implementation (Q)

```
def reverse(s):
    """Returns: reverse of s
    Precondition: s a string"""
    # 1. Handle base case
    if len(s) <= 1:
        return s

    # 2. Break into two parts
    half  = len(s)//2
    left  = reverse(s[:half])
    right = reverse(s[half:])

    # 3. Combine the result
    return right+left
```

Does this work?

A: **YES**

B: **NO**

# Alternate Implementation (A)

```
def reverse(s):
    """Returns: reverse of s
    Precondition: s a string"""
    # 1. Handle base case
    if len(s) <= 1:
        return s

    # 2. Break into two parts
    half  = len(s)//2
    left  = reverse(s[:half])
    right = reverse(s[half:])

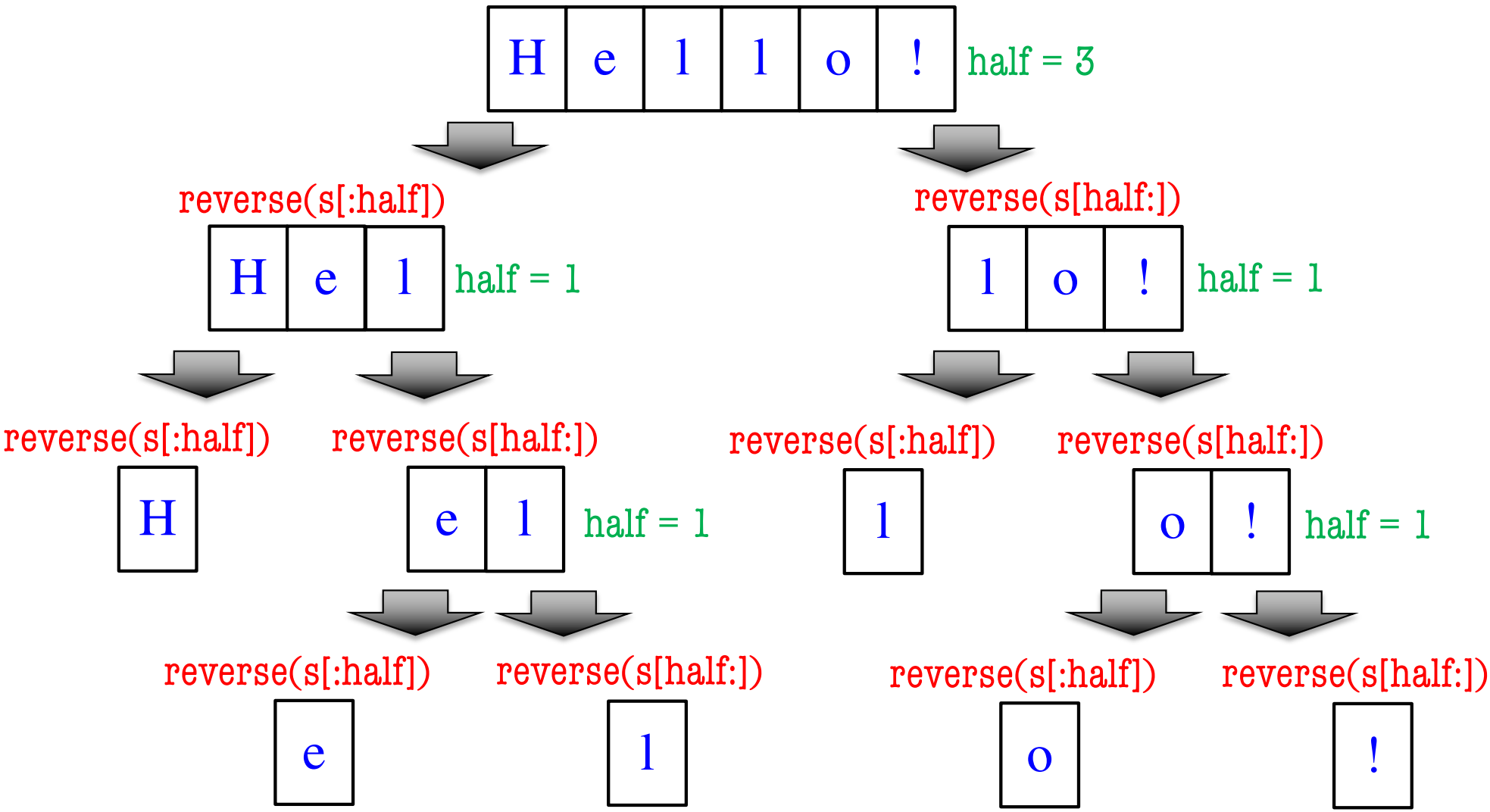
    # 3. Combine the result
    return right+left
```

Does this work?

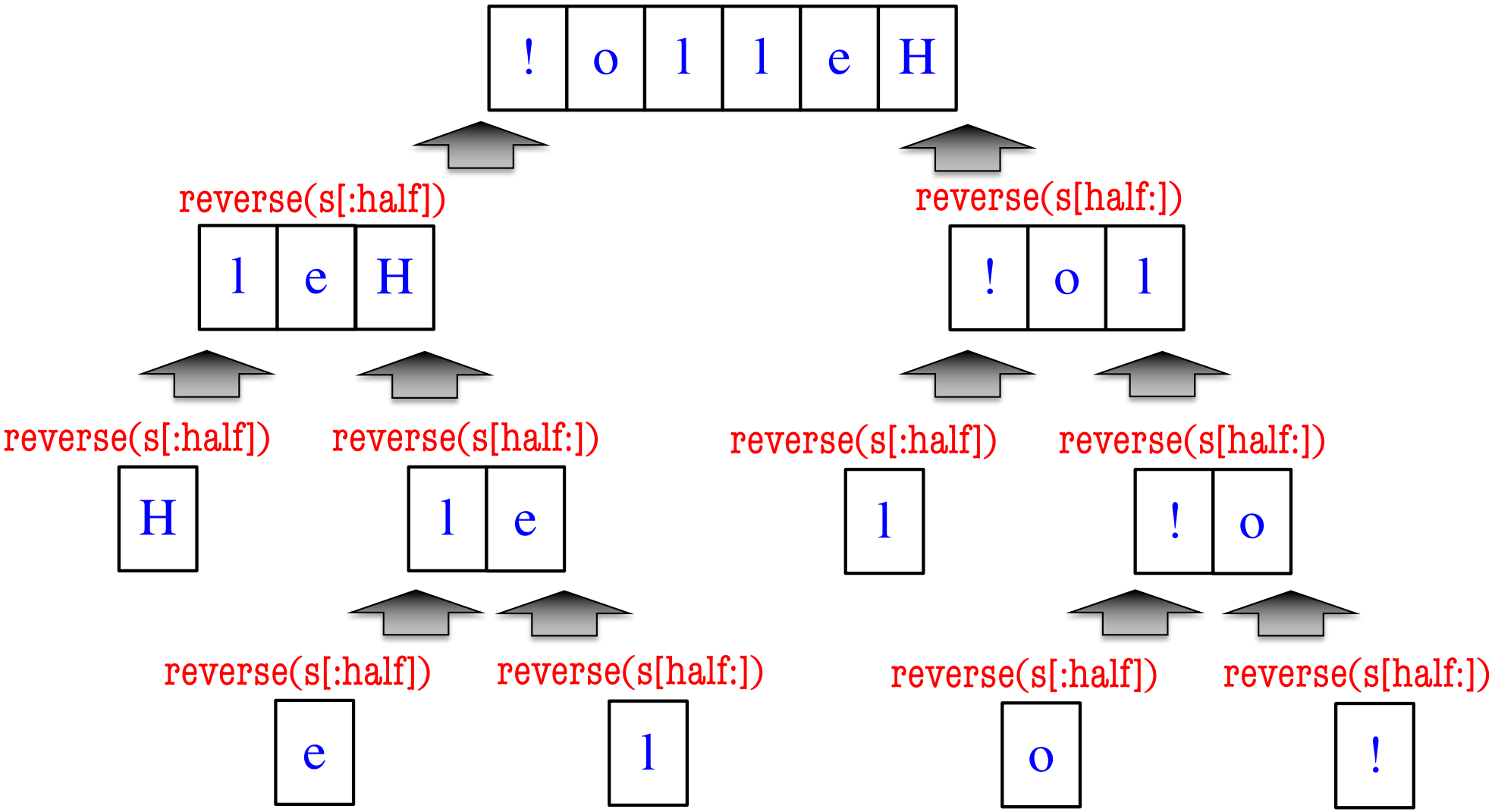
**CORRECT** A: **YES**

B: **NO**

# Alternate Implementation



# Alternate Implementation



# Example: Palindromes

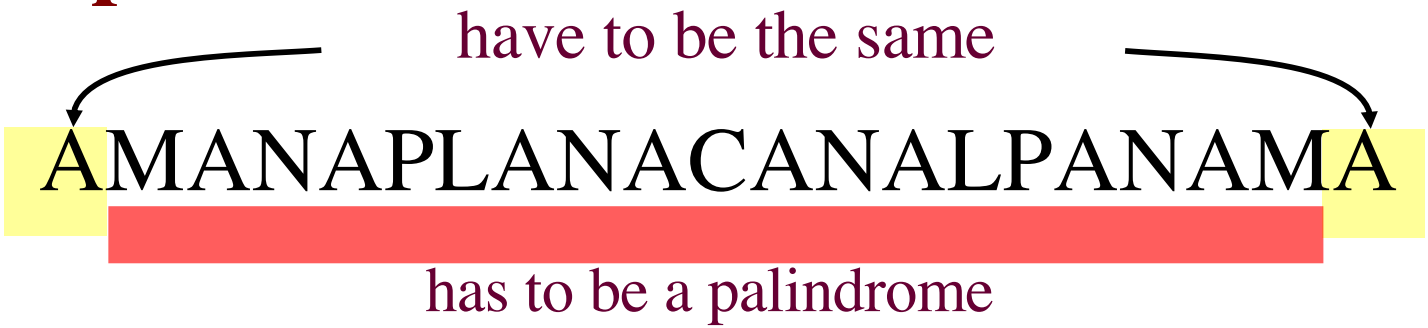
- **Example:**

AMANAPLANACANALPANAMA

- Can we define recursively?

# Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are equal, and
  - the rest of the characters form a palindrome
- **Example:**




- **Implement:** `def ispalindrome(s):`

```
    """Returns: True if s is a palindrome"""
```

# Example: Palindromes

String with ≥ 2 characters is a palindrome if:

- its first and last characters are equal, and
- the rest of the characters form a palindrome

Recursive Definition

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
    if len(s) < 2:
        return True

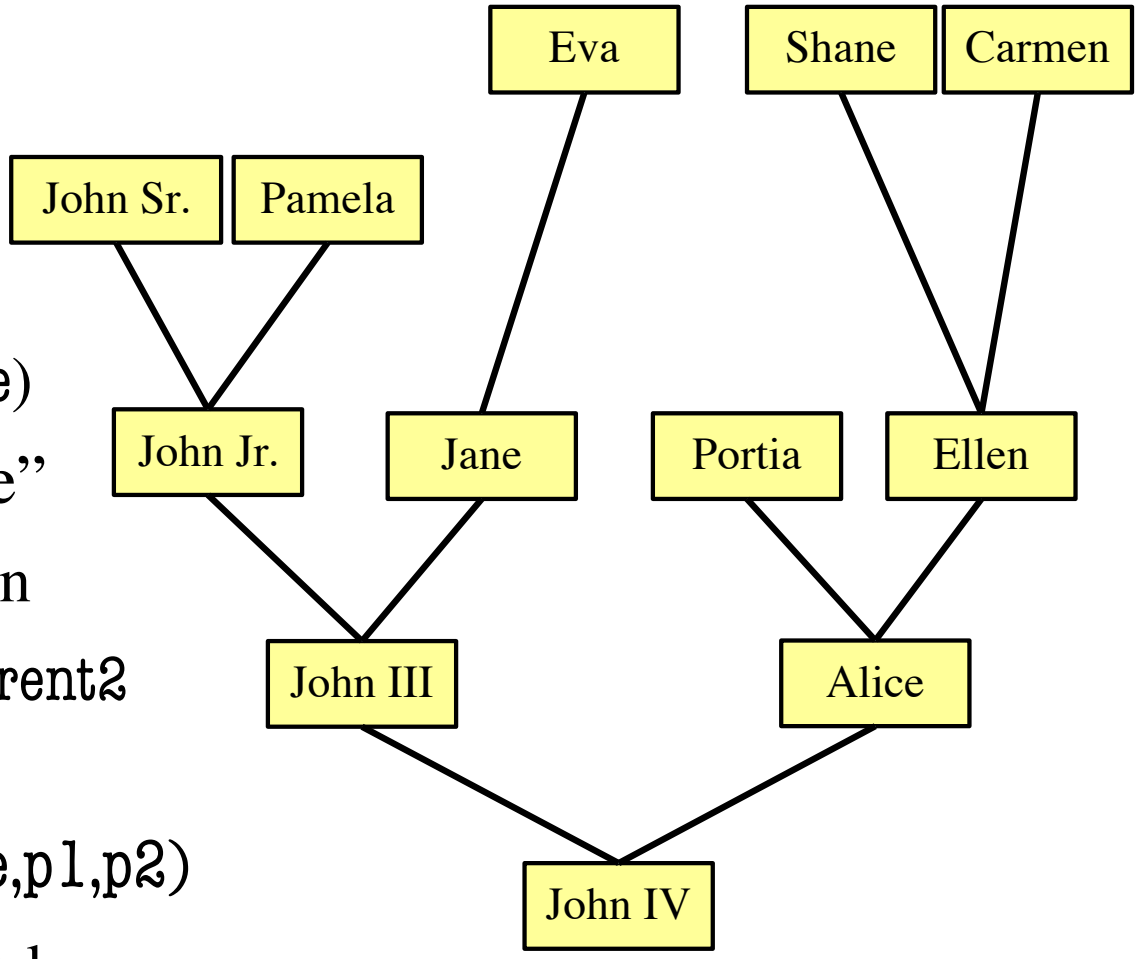
    ends = s[0] == s[-1]
    middle = ispalindrome(s[1:-1])
    return ends and middle
```

**Base case**

**Recursive case**

# Recursion and Objects

- Class Person
  - Objects have 3 attributes
  - `name`: String
  - `parent1`: Person (or `None`)
  - `parent2`: Person (or `None`)
- Represents the "family tree"
  - Goes as far back as known
  - Attributes `parent1` and `parent2` are `None` if not known
- **Constructor**: `Person(name,p1,p2)`
  - Or `Person(n)` if no parents known



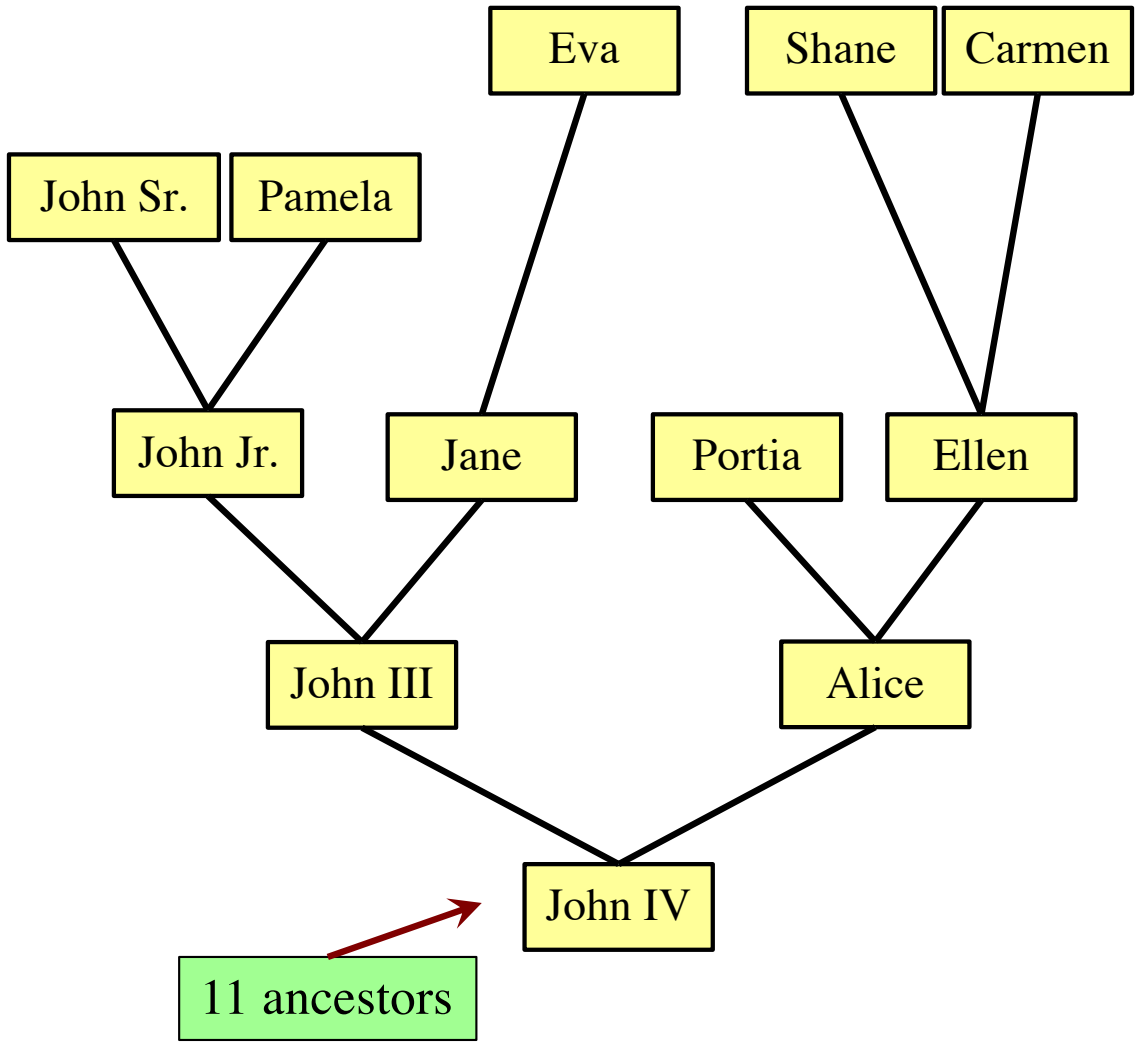
# Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    # No parents
    # (no ancestors)


    # 2. Break into two parts
    # Has parent1 or parent2
    # Count ancestors of each one
    # (plus parent1, parent2 themselves)



    # 3. Combine the result
```

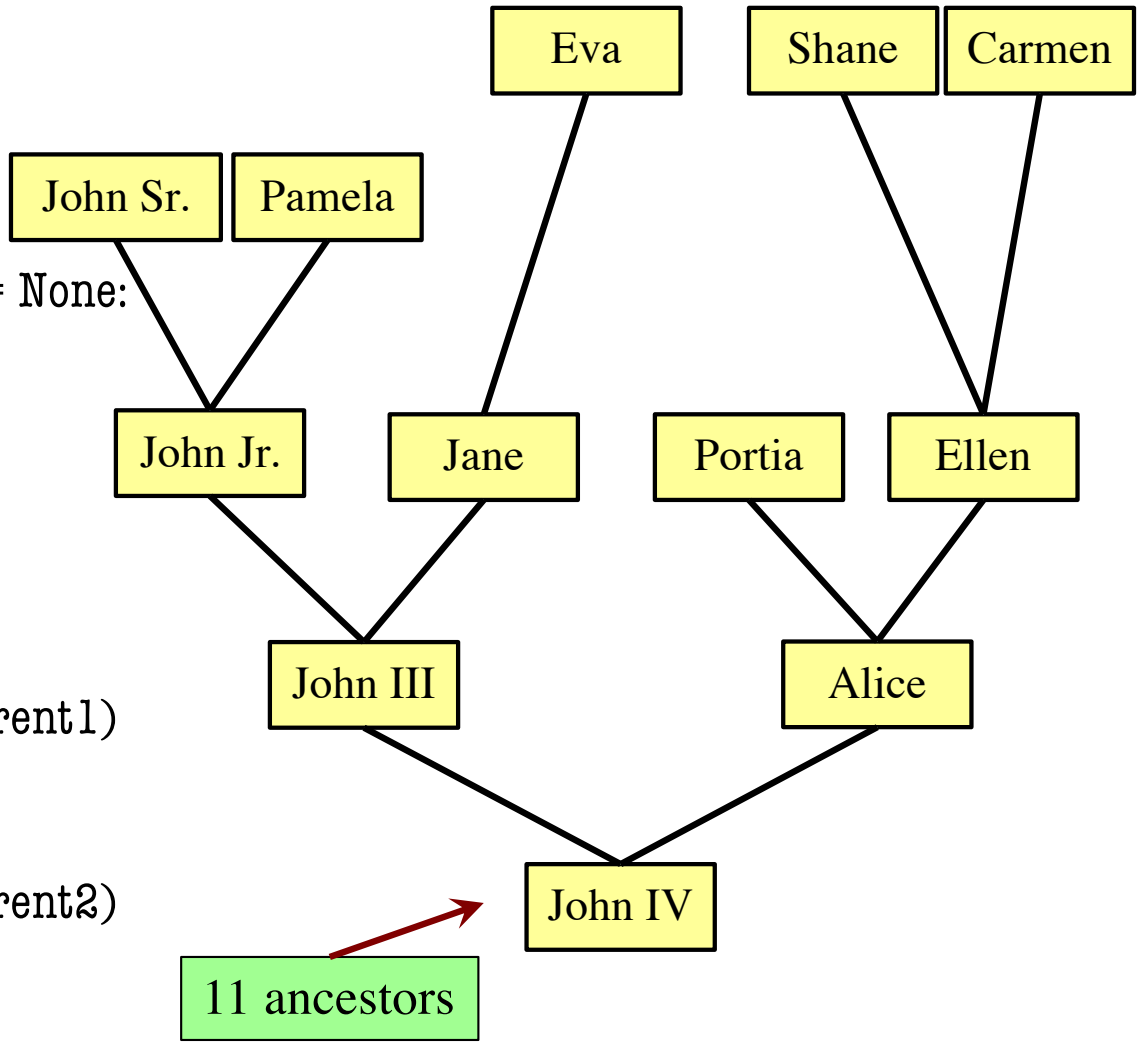


# Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    if p.parent1 == None and p.parent2 == None:
        return 0


    # 2. Break into two parts
    parent1s = 0
    if p.parent1 != None:
        parent1s = 1+num_ancestors(p.parent1)
    parent2s = 0
    if p.parent2 != None:
        parent2s = 1+num_ancestors(p.parent2)

    # 3. Combine the result
    return parent1s+parent2s
```



# Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle base case.
    if p.parent1 == None and p.parent2 == None:
        return 0

    # 2. Break into two parts
    parent1s = 0
    if p.parent1 != None:
        parent1s = 1+num_ancestors(p.parent1s)
    parent2s = 0
    if p.parent2 != None:
        parent2s = 1+num_ancestors(p.parent2s)

    # 3. Combine the result
    return parent1s+parent2s
```

We don't actually need this.
It is handled by the conditionals in #2.

# Challenge: All Ancestors

```
def all_ancestors(p):
    """Returns: list of all ancestors of p"""
    # 1. Handle base case.
    # 2. Break into parts.
    # 3. Combine answer.
```