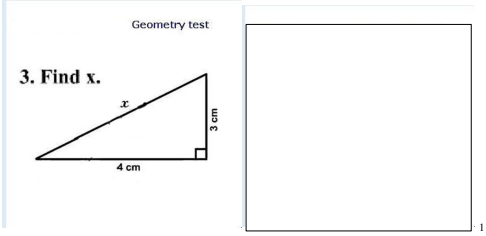
We develop recursive functions and look at execution of recursive functions

CS1110 2 Mar 2010
More on Recursion

Study Sect 15.1, p. 415. Watch activity 15-2.1 on the CD. In DrJava, write and test as many of the self-review exercises as you can (disregard those that deal with arrays).



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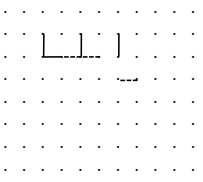
A game



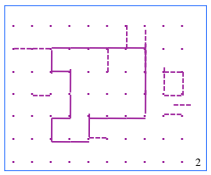
while there is room
A draws — or | ;
B draws --- or ¦ ;

A and B alternate moves

A wants to get a solid closed curve.
B wants to stop A from getting a solid closed curve.
Who can win? What strategy to use?

Board can be any size: m by n dots, with m > 0, n > 0

A won the game to the right because there is a solid closed curve.



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Announcements

- Bring iclickers on Thursday. They will be used
- A3 is graded. Request a regrade ONLY on the CMS. You must explain what you think was graded unfairly or wrong

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```
/** =  non-negative n, with commas every 3 digits
       e.g. commafy(5341267) = "5,341,267" */
public static String commafy(int n) {

}
```

What is the base case?
A: 0..1
B: 0..9
C: 0..99
D: 0..999
E: 0..9999

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```
/** =  non-negative n, with commas every 3 digits
       e.g. commafy(5341267) = "5,341,267" */
public static String commafy(int n) {
    1: if (n < 1000)
        2: return "" + n;
    // n >= 1000
    3: return  commafy(n/1000) +  "," + to3(n%1000);
}
/** = p with at least 3 chars —
      0's prepended if necessary */
public static String to3(int p) {
   if (p < 10) return "00" + p;
   if (p < 100) return "0" + p;
   return  "" + p;
}
```

Executing recursive function calls.

commafy(5341266 + 1)

commafy: 1 | Demo

n

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Recursive functions

Properties:

(1) $b^c = b * b^{c-1}$

(2) For c even

$b^c = (b*b)^{c/2}$

e.g 3*3*3*3*3*3*3*3
= (3*3)*(3*3)*(3*3)*(3*3)

```
/** = b^c. Precondition: c ≥ 0*/
public static int exp(double b, int c)
```

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Recursive functions

```
/** = b^c. Precondition: c ≥ 0*/
public static int exp(double b, int c) {
   if (c = 0)
      return 1.0;
   if (c is odd)
      return b * exp(b, c–1);
   // c is even and > 0
   return exp(b*b, c / 2);
}
```

c	number of calls
0	1
1	2
2	2
4	3
8	4
16	5
32	6
2^n	n + 1

32768 is 2^{15}

so b^{32768} needs only 16 calls!

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Binary arithmetic

Decimal	Binary	Octal
00	00	00
01	01	01
02	10	02
03	11	03
04	100	04
05	101	05
06	110	06
07	111	07
08	1000	10
09	1001	11
10	1010	12

	Binary
$2^0 = 1$	1
$2^1 = 2$	10
$2^2 = 4$	100
$2^3 = 8$	1000
$2^4 = 16$	10000
$2^5 = 32$	100000
$2^6 = 64$	1000000
$2^{15} = 32768$	1000000000000000

Test c odd: Test last bit = 1

Divide c by 2: Delete the last bit

Subtract 1 when odd: Change last bit from 1 to 0.

Exponentiation algorithm processes binary rep. of the exponent.

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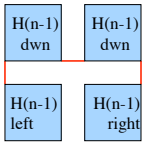
Hilbert's space-filling curve

Hilbert(1):

Hilbert(2):

Hilbert(n):



As the size of each line gets smaller and smaller, in the limit, this algorithm fills every point in space. Lines never overlap.

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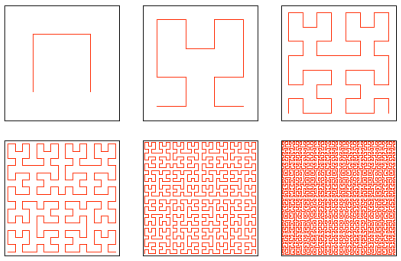
Hilbert's space-filling curve



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