Graph problems

Prof. Ramin Zabih

http://cs100r.cs.cornell.edu

Cornell University
Computer Science

Administrivia

- Assignment 2 is due Friday
  - Working Roomba’s are on the way (?)
- Quiz 3 on Tuesday 9/25
  - Coverage through next lecture
- Optional lecture topics are ready
  - First one will be RDZ talking about graphs
    - Graph questions are favorites for interviewers (at places like Google and Microsoft)

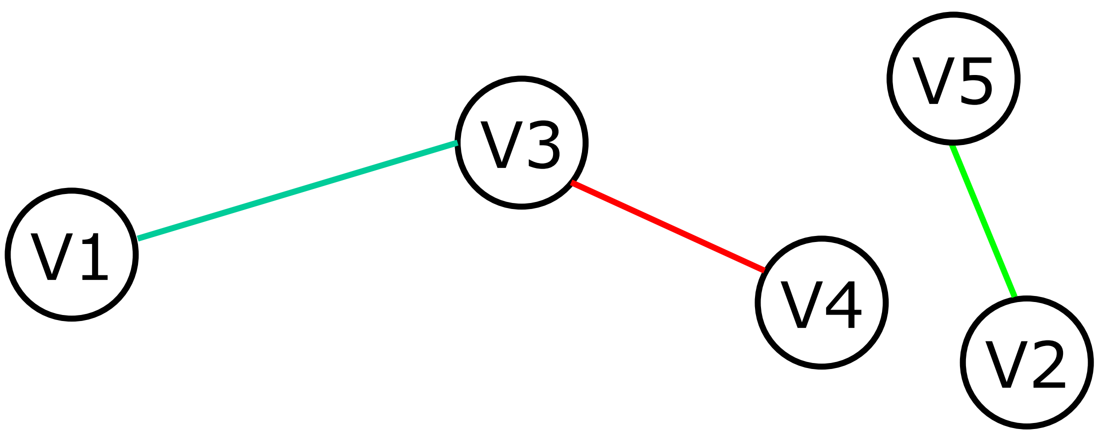
Some major graph problems

- Graph coloring
  - Ensuring that radio stations don't clash
- **Graph connectivity**
  - How fragile is the internet?
- Graph cycles
  - Helping FedEx/UPS/DHL plan a route
- Planarity testing
  - Connecting computer chips on a motherboard
- Graph isomorphism
  - Is a chemical structure already known?

How to represent graphs?

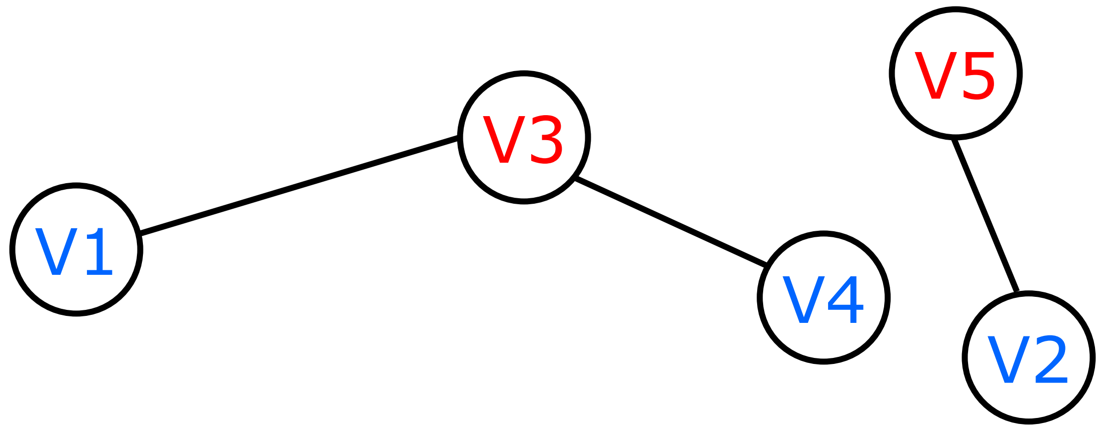
- The adjacency matrix A is n by n
  - We will give each vertex a number 1..n
  - A(r,c) == 1 when there is an edge between vertex r and vertex c
  - Note that A(r,c) == A(c,r)

0	0	1	0	0
0	0	0	0	1
1	0	0	1	0
0	0	1	0	0
0	1	0	0	0



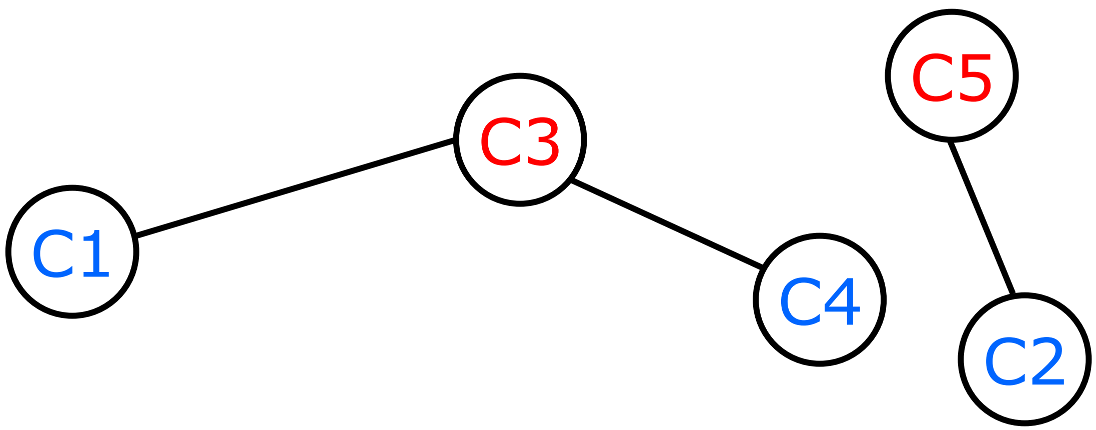
Graph coloring problem

- Given a graph and a set of colors {1..k}
- We want to assign each vertex a color
  - Two adjacent vertices have different colors



Radio frequencies via coloring

- Make a graph where a station is a vertex
  - Put an edge between two stations that clash
    - I.e., if their signal areas overlap
  - Any coloring is a non-clashing layout
    - Can you prove this? What about vice-versa?



Verifying vs. finding

- One theme we've seen is that it can be easy to **verify** you have the right answer
  - I.e., prove that your candidate is the solution
  - Example: sorting
- If this is easy, there is a dumb algorithm
  - Guess and verify
  - Or: try everything
- It's not always easy to verify that an answer is correct
  - Example: is this graph 4-colorable?

Graphs and paths

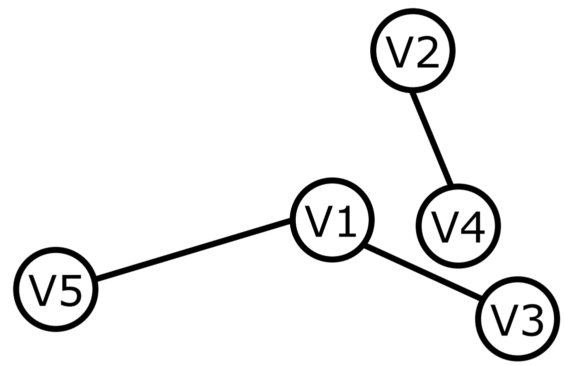
- Can you get from vertex V to vertex W?
  - Is there a route from one city to another?
- More precisely, is there a sequence of vertices $\{V, V_1, V_2, \ldots, V_k, W\}$ such that every adjacent pair has an edge between them
  - This is called a **path**
  - A **cycle** is a path from V to V
  - A path is **simple** if no vertex appears twice

Graph connectivity

- For any pair of nodes, is there a path between them?
  - Basic idea of the Internet: you can get from any computer to any other computer
  - This pair of nodes is called *connected*
    - A set of nodes is connected if all pairs are
  - A graph is connected if all nodes are connected
- Related question: if I remove an arbitrary node, is the graph still connected?
  - Is the Internet intact if any 1 computer fails?

Connected components

- Even if all nodes are not connected, there will be subsets that are all connected
  - Connected components



  - All nodes are connected
  - Is this sufficient? Why not?

Blobs are components!

A	0	0	0	0	0	0	0	B	0
0	0	0	0	0	0	0	0	C	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	D	0	0	0	0	0
0	0	0	E	F	G	0	0	0	0
0	0	0	H	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0