We derive recursive functions and look at execution of recursive calls.

CS100J 2 Oct 2006
More on Recursion

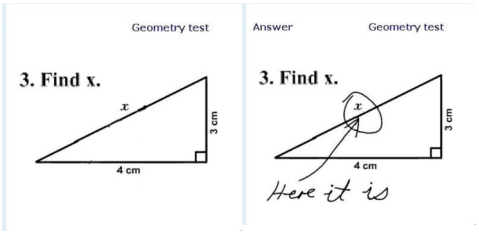
Study Sect 15.1, p. 415. Watch activity 15-2.1 on the CD. In DrJava, write and test as many of the self-review exercises as you can (disregard those that deal with arrays).

My first job was working in an orange juice factory, but I got canned: couldn't concentrate.
Then I worked in the woods as a lumberjack, but I just couldn't hack it, so they gave me the axe.
After that I tried to be a tailor, but I just wasn't suited for it. Mainly because it was a so-so job.
Next I tried working in a muffler factory but that was exhausting.
I worked as a pilot but eventually got grounded for taking off too much.
Then I tried teaching but I couldn't make the grade.

Get more of these from the course website

1

Geometry test



2

Recursive functions

/** = a copy of s in which s[0..1] are swapped, s[2..3] are swapped, s[3..4] are swapped, etc. */

public static String swapAdjacent(String s)

/** = b^c. Precondition: $c \geq 0$*/
public static int exp(**int** b, **int** c)

Properties:

(1) $b^c = b * b^{c-1}$

(2) For c even

$b^c = (b*b)^{c/2}$

e.g 3*3*3*3*3*3*3*3

= (3*3)*(3*3)*(3*3)*(3*3)

3

Recursive functions

/** = b^c. Precondition: $c \geq 0$*/
public static int exp(**int** b, **int** c) {
  if (c = 0)
    return 1;
  if (c is odd)
    return b * exp(b, c–1);
  // c is even and > 0
  return exp(b*b, c / 2);
}

c	number of calls
0	1
1	2
2	2
4	3
8	4
16	5
32	6
2^n	n + 1

32768 is 2^{15}
so b^{32768} needs only 16 calls!

4

Binary arithmetic

Decimal	Binary	Dec	Binary
00	00	$2^0 = 1$	1
01	01	$2^1 = 2$	10
02	10	$2^2 = 4$	100
03	11	$2^3 = 8$	1000
04	100	$2^4 = 16$	10000
05	101	$2^5 = 32$	100000
06	110	$2^6 = 64$	1000000
07	111	$2^{15} = 32768$	1000000000000000
08	1000		
09	1001		
10	1010		

Test c odd: Test last bit = 1

Divide c by 2: Delete the last bit

Subtract 1 when odd: Change last bit from 1 to 0.

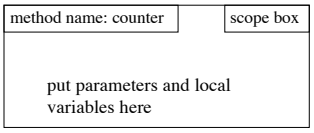
Exponentiation algorithm processes the binary representation of the exponent.

5

Executing recursive calls

Steps in executing a call:

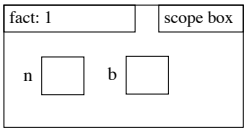
1. Draw a frame for the call, including the parameters and local variables and scope box.
2. Assign argument values to the parameters.
3. Executed the method body.
4. Erase the frame —and give value of function call to caller.



6

Executing recursive calls

```
/** = n!. Precondition: n ≥ 0 */
public static int fact(int n) {
  if (n <= 1)
    return 1;
  int b = fact(n-1);
  return n * b;
}
```



0! = 1. n! = n * (n-1) * (n-2) * … * 2 * 1

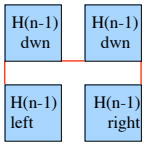
1. Draw frame
2. Assign argument values to parameters
3. Execute body
4. Erase frame —and give value of function call back to caller.

Hilbert's space-filling curve

Hilbert(1):

Hilbert(2):

Hilbert(n):



As the size of each line gets smaller and smaller, in the limit, this algorithm fills every point in space. Lines never overlap.

Hilbert's space-filling curve