CS100J 02 May
Matlab and PI and other things

- The notation `j:k` gives a row matrix consisting of the integers from `j` through k.

```
>> 1:8
ans =
     1     2     3     4     5     6     7     8
```

- The notation `j:b:k` gives a row matrix consisting of the integers from `j` through k in steps of b.

```
 >> 1:2:10
ans =
     1     3     5     7     9
```

- To get a vector of n linearly spaced points between lo and hi, use `linspace(lo,hi,n)`

```
>> linspace(1,3,5)
ans =
    1.0    1.5    2.0    2.5    3.0
```

- To transpose a matrix m, write m'

```
>> (1:3)'
ans = 1
      2
      3
```


sum, prod, cumsum, cumprod

- Function `sum` adds row elements, and function `prod` multiplies row elements:

```
>> sum(1:1000)
ans =
     500500
>> prod(1:5)
ans =
     120
```

- Function `cumsum` computes a row of partial sums and cumprod computes a row of partial products:

```
>> cumprod(1:7)
ans =
    1    2    6    24    120    720
  5040
>> cumsum(odds)
ans =
     1    4    9    16    25    36
  49    64
```


Compute Pi by Euler Formula

- Leonard Euler (1707-1783) derived the following infinite sum expansion:

$$\pi^2 / 6 = \Sigma\ 1/j^2 \quad \text{(for j from 1 to } \infty\text{)}$$

```
>> pi = sqrt( 6 .* cumsum(1 ./ (1:10) .^ 2));
>> plot(pi)
```

- To define a function, select **New/m-file** and type definition:

```
% = a vector of approximations to pi.
function e = euler(n)
 e= sqrt( 6 .* cumsum(1 ./ (1:n) .^ 2));
```

- Select SaveAs and save to a file with the same name as the function.

- To invoke:

```
>> pi= euler(100);
>> plot(pi)
```


Help System

- Use on-line help system

```
 >> help function
 ... description of how to define
functions ...

 >> help euler
 = a vector of approximations to pi,
using Euler's appoximation
```


Compute Pi by Wallis Formula

- John Wallis (1616-1703) derived the following infinite sum expansion:

$$\pi / 2 = \frac{(2*2) * (4*4) * (6*6) * \ldots}{(1*3) * (3*5) * (5*7) * \ldots}$$

- Terms in Numerator

```
 evens .^ 2
```

- Terms in Denominator

```
  1  3   5   7    9 ...   odds
  3  5   7   9   11 ...   odds + 2
  ----------------
1*3 3*5 5*7 7*9  9*11 . . .
```

  i.e. `odds .* (odds + 2)`

- Quotient

```
prod( (evens .^ 2) ./ (odds .*
(odds+2))
```

- Successive approximations to Pi

```
pi = 2 .* cumprod( (evens.^2) ./
(odds .* (odds+2)) )
```


Wallis Function

- Function Definition

```
function w = wallis(n)
% compute successive approx's to pi.
evens = 2 .* (1:n);
odds = evens - 1;
odds2 = odds .* (odds + 2);
w = 2 .* cumprod( (evens .^ 2) ./ odds2
  );
```

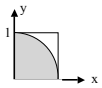
- Contrasting Wallis and Euler approximations

```
>> plot(1:100, euler(100), 1:100,
  wallis(100))
```

Compute Pi by Throwing Darts

- Throw random darts at a circle of radius 1 inscribed in a 2-by-2 square.

- The fraction hitting the circle should be the ratio of the area of the circle to the area of the square: f = π / 4
- This is called a Monte Carlo method



Darts

- (h,w) yields an h-by-w matrix of random numbers between 0 and 1.

```
>> x = rand(1,10);
>> y = rand(1,10);
```

- Let `d2` be the distance squared from the center of the circle.

```
>> d2 = (x .^ 2) + (y .^ 2);
```

- `in` be a row of 0's and 1's signifying whether the dart is in (1) or not in (0) the circle. Note 1 is used for **true** and 0 for **false**.

```
>> in = d2 <= 1;
```

- `hits(i)` be the number of darts in circle in i tries

```
>> hits = cumsum(in);
```

- `f(i)` be franction of darts in circle in i tries

```
>> f = hits ./ (1:10);
```

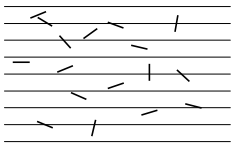
- `pi` be successive approximations to pi

```
>> pi = 4 .* f;
```


Compute Pi by Throwing Needles

- In 1777, Comte de Buffon published this method for computing π:

  N needles of length 1 are thrown at random positions and random angles on a plate ruled by parallel lines distance 1 apart. The probability that a needle intersects one of the ruled lines is 2/π. Thus, as N approaches infinity, the fraction of needles intersecting a ruled line approaches 2/π.



Subscripting

- Subscripts start at 1, not zero.

```
>> a = [1 2 3; 4 5 6; 7 8 9]
ans = 1    2    3
      4    5    6
      7    8    9
>> a(2,2)
ans =
     5
```

- A range of indices can be specified:

```
>> a(1:2, 2:3)
ans =
     2    3
     5    6
```

- A colon indicates all subscripts in range:

```
>> a(:, 2:3)
ans =
     2    3
     5    6
     8    9
```


Control Structures: Conditionals

```
if expression
   list-of-statements
end

if expression
   list-of-statements
else
 list-of-statements
end

if expression
   list-of-statements
elseif expression
 list-of-statements
 . . .
elseif expression
 list-of-statements
else
 list-of-statements
end
```


Control Structures: Loops

```
while expression
   list-of-statements
end


for variable = lo:hi
   list-of-statements
end


for variable = lo:by:hi
   list-of-statements
end
```