CS100J 28 Nov 2006
Matlab and PI and array operations

Did you complete your course evaluations?

From the online unabridged Myriam-Webster:

logorrhea: pathologically excessive and often incoherent talkativeness.

Based on that, I coined
digirrhea: what happens when I type faster than I think.

I don't require novocaine when I go the the dentist because I
transcend dental medication

1

Creating arrays

- `j:k` gives a row matrix consisting of the integers from `j` through k.

```
>> 1:8
ans =
     1    2    3    4    5    6    7    8
```

- `j:b:k` gives a row matrix consisting of the integers from `j` through k in steps of b.

```
>> 1:2:10
ans =
     1    3    5    7    9
```

- `linspace(lo,hi,n)` gives a vector of n linearly spaced points between lo and hi

```
>> linspace(1,3,5)
ans =
      1.0    1.5    2.0    2.5    3.0
```

- To transpose a matrix m, write m'

```
>> (1:3)'
ans = 1
      2
      3
```

2

sum, prod, cumsum, cumprod

- Function `sum` adds row elements, and function `prod` multiplies row elements:

```
>> sum(1:1000)
ans =
     500500
>> prod(1:5)
ans =
     120
```

- Function `cumsum` computes a row of partial sums and cumprod computes a row of partial products:

```
>> cumprod(1:7)
ans =
     1  2  6  24  120  720   5040
>> cumsum(odds)
ans =
     1   4   9   16   25   36   49   64
```

3

Compute Pi by Euler Formula

- Leonard Euler (1707-1783) derived the following infinite sum expansion:

$\pi^2 / 6 = \Sigma\ 1/j^2$ (for j from 1 to ∞)

```
>> pi = sqrt( 6 .* cumsum(1 ./ (1:10) .^ 2));
>> plot(pi)
```

- Define a function, select **New/m-file**

```
% = a vector of approximations to pi.
function e = euler(n)
 e= sqrt( 6 .* cumsum(1 ./ (1:n) .^ 2));
```

- Select SaveAs and save to a file with the same name as the function.

- To invoke:

```
>> pi= euler(100);
>> plot(pi)
```

4

Help System

- Use on-line help system:

```
 >> help function
 ... description of how to define functions ...
```

- Use the help system to see what a function does:

```
 >> help euler
 = a vector of approximations to pi, using Euler's appoximation
```

5

Compute Pi by Wallis Formula

- John Wallis (1616-1703) :

```
      (2*2) * (4*4) * ...
π/2 = --------------------------
      (1*3) * (3*5) * ...
```

- Terms in Numerator

```
evens .^ 2
```

- Terms in Denominator

```
 1 3  5  7   9 ...
 3 5  7  9  11 ...
-----------------
1*3 3*5 5*7 7*9  9*11 ...
```

i.e. `odds .* (odds + 2)`

- Quotient

```
prod( (evens .^ 2) ./
      (odds .* (odds+2))
```

- Successive approximations to Pi

```
pi= 2 .* cumprod( (evens.^2) ./
      (odds .* (odds+2)) )
```

6

Wallis Function

- Function Definition

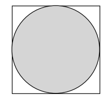
```
function w = wallis(n)
% compute successive approx's to
  pi.
evens= 2 .* (1:n);
odds= evens - 1;
odds2= odds .* (odds + 2);
w= 2 .* cumprod(
          (evens .^ 2) ./ odds2 );
```

- Contrasting Wallis and Euler approximations

```
>> plot(1:100, euler(100),
        1:100, wallis(100))
```

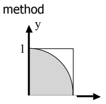

Compute Pi by Throwing Darts

- Throw random darts at a circle of radius 1 inscribed in a 2-by-2 square.



- The fraction hitting the circle = ratio of area of circle to area of square:
  $f = \pi / 4$

- " Monte Carlo" method



Darts

- matrix of random numbers between 0 and 1.

```
>> x= rand(1,10);
>> y= rand(1,10);
```

- `d2`: distance squared from center of circle.

```
>> d2= (x .^ 2) + (y .^ 2);
```

- `in`: a row of 0's and 1's signifying whether the dart is in (1) or not in (0) the circle. Note: 1 is used for **true** and 0 for **false**.

```
>> in= d2 <= 1;
```

- `hits(i)`: number of darts in circle in i tries

```
>> hits= cumsum(in);
```

- `f(I)`: franction of darts in circle in i tries

```
>> f= hits ./ (1:10);
```

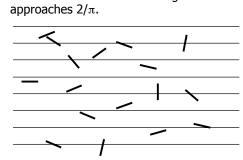
- `pi`: successive approximations to pi

```
>> pi= 4 .* f;
```


Compute Pi by Throwing Needles

- 1777, Comte de Buffon's method for computing π:

  Throw N needles of length 1 at random positions and random angles on a plate ruled by parallel lines distance 1 apart. Probability that a needle intersects one of the ruled lines is $2/\pi$. So, as N approaches infinity, the fraction of needles intersecting a ruled line approaches $2/\pi$.



Subscripting

- Subscripts start at 1, not zero.

```
>> a= [1 2 3; 4 5 6; 7 8 9]
ans=  1    2    3
      4    5    6
      7    8    9
>> a(2,2)
ans = 5
```

- A range of indices can be specified:

```
>> a(1:2, 2:3)
ans =   2    3
        5    6
```

- A colon indicates all subscripts in range:

```
>> a(:, 2:3)
ans =
     2    3
     5    6
     8    9
```