

**CS100J 31 October 2006**  
**Arrays: searching & sorting. Reading: 8.5**

Searching and sorting algorithms are on the course website.

**Merry Christmas!! On Halloween?**

Decimal	Octal	Binary	Decimal	Octal	Binary	Decimal	Octal	Binary
00	00	0000	11	13	01011	22	26	010110
01	01	0001	12	14	01100	23	27	010111
02	02	0010	13	15	01101	24	30	011000
03	03	0011	14	16	01110	25	31	011001
04	04	0100	15	17	01111	26	32	011010
05	05	0101	16	20	10000	27	33	011011
06	06	0110	17	21	10001	28	34	011100
07	07	0111	18	22	10010	29	35	011101
08	10	1000	19	23	10011	30	36	011110
09	11	1001	20	24	10100	31	37	011111
10	12	1010	21	25	10101	32	40	100000

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Decimal 5482:  $5 \cdot 10^3 + 4 \cdot 10^2 + 8 \cdot 10^1 + 2 \cdot 10^0$

Octal 3726:  $3 \cdot 8^3 + 7 \cdot 8^2 + 2 \cdot 8^1 + 6 \cdot 8^0$

Binary 1011:  $1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

*/\*\* = a string that contains the binary representation of n.*

*Precondition: n >= 0 \*/*

```
public static String binary(int n) {
    if (n <= 1)
        return "" + n;

    return binary(n/2) + (n%2);
}
```

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Decimal 5482:  $5 \cdot 10^3 + 4 \cdot 10^2 + 8 \cdot 10^1 + 2 \cdot 10^0$

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Binary 1011:  $1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

*/\*\* = a string that contains the binary representation of n.*

*Precondition: n >= 0 \*/*

```
public static String binary(int n) {
    if (n == 0) return "0";
    String res= ""; int k= n;
    // binary repr of n = binary repr of k + res;
    while (k > 0) {
        res= (k%2) + res; k= k / 2;
    }
    // binary repr of n = res
    return res;
}
```

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**Getting an invariant as picture:**

**Combine pre- and post-condition**

pre: 

0	?	n
---	---	---

post: 

0	n
reds	whites blues

*Dutch national flag*

inv: 

0	h	k	j	n
reds	whites	?	blues	

*Finding the minimum value of an array*

pre: 

0	?	n
---	---	---

post: 

0	n
	x is the min of this

inv: 

0	k	n
x is the min of this	?	

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Binary search: Find a position in sorted array b that satisfies

$b[0..k] \leq x$  and  $b[k+1..b.length-1]$

pre: 

0	?	b.length
---	---	----------

post: 

0	k	b.length
<= x	> x	

What invariant can you use?

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• **Linear search.** *Vague spec.:* find first occurrence of v in b[h..k-1].

*Better spec.:* Store an integer in i to truthify:

*postcondition:* (0) v is not in b[h..i-1]

(1) Either i = k or v = b[k]

*invariant:* v is not in b[h..i-1]

• **Finding the min.** *Vague spec.:* Find the min of b[h..k]

*Better spec.:* *Precondition:* h <= k (because an empty set of values has no min)

Store in i to truthify:

*postcondition:* b[m] is the min of b[h..k] (and it is first occurrence of the min)

*invariant:* b[m] is the min of b[h..t-1] (and it is first occur. of the min)

• **Binary search:** *Vague spec:* Look for v in sorted array segment b[h..k].

*Better spec:*

*Precondition:* b[h..k] is sorted (in ascending order).

Store in i to truthify:

*postcondition:* b[h..i] <= v and v < b[i+1..k]

*invariant:* b[h..i] <= v and v < b[j..k]

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• **Dutch national flag.** *Vague spec.:*  $b[0..n-1]$  contains only red, white, blue balls.  
 Sort it using only swaps.

**Better spec.:** *Precondition:*  $n \geq 0$   
 Permute  $b[0..n-1]$  to truthify:

*postcondition:*  $b[0..h-1]$  are red balls  
 $b[h..k-1]$  are white balls  
 $b[k..n-1]$  are blue balls

*precondition:*

0	?			n
---	---	--	--	---

*postcondition:*

0	h	k	n
reds	whites	blues	

*invariant :*

0	h	k	j	n
reds	whites	?	blues	

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**Sorting:**

*pre:*

h	?		k
---	---	--	---

*post:*

h	sorted		k
---	--------	--	---

**Four possible invariants:**

*pre:*

h	i	k
sorted	?	

*pre:*

h	i	k
sorted	?	

*pre:*

h	i	k
?	sorted	

*pre:*

h	i	k
?	sorted	

Write the algorithm using each one.

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**Sorting:**

*pre:*

h	?		k
---	---	--	---

*post:*

h	sorted		k
---	--------	--	---

*insertionsort inv:*

h	i	k
sorted	?	

**The invariant for algorithm selection sort adds another property to each segment: the first segment contains the smaller values.**

*selectionsort inv:*

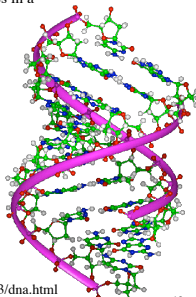
h	i	k
$\leq b[i..k]$ , sorted	$\geq b[h..i-1]$ , ?	

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Deoxyribonucleic acid (DNA) is the building block of all life. Each DNA strand consists of two strings of bases twisted together to form a double helix. The 4 possible bases are represented by G, A, T and C. In a helix, A and T bond together, as do C and G. The two sequences in a helix are complements. For example, these two sequences are complements of each other:

sequence 1:   ACGTTAC  
 sequence 2:   TGCAATG

Paired bases meet at an angle. DNA is a very large molecule; the image shows only a tiny fraction of the typical molecule. For the bacterium *Escherichia coli*, the image would be 80 kilometers long. For a typical piece of DNA from an eukaryote cell, the image would stretch from Dallas to Washington, D. C.! DNA is not fully stretched out inside a cell but is wound around proteins, which protect the DNA.



Taken from [www.ucmp.berkeley.edu/glossary/gloss3/dna.html](http://www.ucmp.berkeley.edu/glossary/gloss3/dna.html)

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