









```
The while loop
x = 0;
                                 To execute the while loop:
x = x + 2*2;
                                 (1) Evaluate condition k != 5;
x = x + 3*3;
                                     if false, stop execution of
x = x + 4*4;
                                     loop.
                                 (2) Execute the repetend.
x=0:
                                 (3) Repeat again from step (1).
int k=2;
                                 Repetend: the thing to be
while ( k != 5) {
                                 repeated. The block:
  x = x + k*k;
                                      {
   k = k + 1;
}
                                 }
```

```
Develop loop to store in x the sum of 1..100.
This time, we'll keep this definition of x and k true:
                 x = sum of h..100
1. How should the loop start? Make range h..100
empty: h= 101; x= 0;
                                                            loopy
2. When can loop stop? What condition lets us
know that x has result? When h == 1
3. How can repetend make progress toward termination? h=h-1;
4. How do we keep def of x, h, k true? x = x + (h - 1):
h= 101: x= 0:
// invariant: x = \text{sum of h..}100
while ( h != 1) {
   x = x + (h - 1);
   h = h - 1;
\frac{1}{x} = \text{sum of } 1..100
```

```
Roach infestation!

/** = number of weeks it takes roaches to fill the apartment --see p 244 of text*/
public static int roaches() {
    double roachVol=.001;  // Space one roach takes
    double aptVol= 20*20*8;  // Apartment volume
    double growthRate= 1.25;  // Population growth rate per week

int w= 0;  // number of weeks

int pop= 100;  // roach population after w weeks

// inv: pop = roach population after w weeks AND

// before week w, volume of the roaches < aptVol
while (aptVol > pop * roachVol ) {
    pop= (int) (pop * growthRate);
    w= w + 1;
    }

return w;
}
```

```
Logarithmic algorithm to
                                              Rest on identities:
  calculate b^{**}c, for c \ge 0
                                              b**0 = 1
/** = b**c, given c \ge 0 */
                                              b^{**}c = b * b^{**}(c-1)
\textbf{public static int} \ exp(\textbf{int} \ b, \textbf{int} \ \ c) \ \{
                                            for even c, b^{**}c = (b^*b)^{**}(c/2)
   if (c == 0) return 1;
                                              3*3 * 3*3 * 3*3 * 3*3 = 3**8
   if (c\%2 = 0) return \exp(b*b, c/2); |_{(3*3)*(3*3)*(3*3)*(3*3)=9**4}
   return b * exp(b, c-1);
                                           Algorithm processes each bit of c
                                           at most twice.
Algorithm processes binary
representation of c
                                           So if c is 2**15 = 32768, algorithm
Suppose c is 14 (1110 in binary)
                                           has at most 2*15 = 30 recursive
1. Test if c is even: test if last bit is 0
                                           Algorithm is logarithmic in c, since
2. To compute c/2 in binary, just
                                           time is proportional to log c
delete the last bit.
```

```
Iterative version of logarithmic algorithm to
calculate b**c,
for c \ge 0 (i.e. b multiplied by itself c times)
/** set z to b**c, given c \ge 0 */
int x=b; int y=c; int z=1;
// invariant: z * x**y = b**c and 0 \le y \le c
while (y != 0) \{
                                      Rest on identities:
   if (y \% 2 == 0)
       \{ x = x * x; y = y/2; \}
                                       b^{**}c = b * b^{**}(c-1)
   else { z=z * x; y=y-1; }
                                       for even c, b^{**}c = (b^*b)^{**}(c/2)
                                       3*3 * 3*3 * 3*3 * 3*3 = 3**8
// \{ z = b**c \}
                                       (3*3)*(3*3)*(3*3)*(3*3) = 9**4
 Algorithm is logarithmic in c, since time is proportional to log c
```

```
Calculate quotient and remainder when dividing x by y x/y = q + r/y \qquad 21/4 = 4 + 3/4 Property: x = q * y + r \text{ and } 0 \le r < y /** Set q to and r to remainder. Note: x > 0 and y > 0 */ int q = 0; int r = x; // invariant: x = q * y + r \text{ and } 0 \le r while (r > y) { r = r - y; q = q + 1; } // \{x = q * y + r \text{ and } 0 \le r < y\}
```