

Backtracking

```
/** = "elements of b can be placed into
 * 2 bags whose sums are equal." */
static boolean split(int[] b) {
    return split(b, 0, 0, 0);
}
```

```
/** Values of b[0..k-1] have been placed into
 * two bags that sum to s1 and s2. Return
 * true iff b[k..] can be placed into the bags
 * so that the bags have the same sum. */
static boolean split(int[] b, int k, int s1, int s2) {
    if (k == b.length) return s1 == s2;
    return split(b, k+1, s1 + b[k], s2)
        || split(b, k+1, s1, s2 + b[k]);
}
```

The call stack contains a frame for the call `split(b)`, with `b` containing a pointer to the array containing five 1's.

`split: b` → {1, 1, 1, 1, 1}

We execute the call that is in the body of `split`. A frame for the call is placed on the call stack and the argument values are stored in the parameters. In the frame for the call, we show parameters `k`, `s1`, and `s2`. We don't show `b`. It is a pointer to the array with five 1's.

<code>split: k, s1, s2</code>
<code>split: 0, 0, 0</code>
<code>split: b</code>

Now execute the body of `split`. Start with the if-statement. Since `k` is 0 and `b.length` is 5, execution of the if-statement terminates. Next, execute the return statement. We evaluate the first call on `split`, putting a frame for the call on the call stack. In the new frame, `k` is 1, which means that `b[0]` has been placed in a bag. Parameter `s1` is 1, indicating that `b[0]` has been placed in the first bag, and the second bag is still empty.

<code>split: 1, 1, 0</code>
<code>split: 0, 0, 0</code>
<code>split: b</code>

Next, execute the body of `split`. This works as before. The if-statement has no effect, and the return statement is executed, causing another frame for the call to be placed on the call stack. This time, `k` is 2 and the first bag contains two values. In fact, we can see that this will happen three more times, until `k` in the top frame is 5. All five values have been placed in the first bag!

<code>split: 5, 5, 0</code>
<code>split: 4, 4, 0</code>
<code>split: 3, 3, 0</code>
<code>split: 2, 2, 0</code>
<code>split: 1, 1, 0</code>
<code>split: 0, 0, 0</code>
<code>split: b</code>

Backtracking

At this point, `k = 5` in the frame for the call, so `k = b.length`, and the if-condition of the if-statement in the body of `split` is true. Parameter `s1` is not equal to `s2`, so the frame for the call is popped and false is returned. This is the first example of *backtracking*; a choice was made when `k` was 4, and it didn't work.

So, the second choice, putting `b[k]` into the second bag, is tried. We make the frame where `k` is 4 red to indicate that the second call on `split` in the return statement is being executed.

<code>split: 5, 4, 1</code>
<code>split: 4, 4, 0</code>
<code>split: 3, 3, 0</code>
<code>split: 2, 2, 0</code>
<code>split: 1, 1, 0</code>
<code>split: 0, 0, 0</code>
<code>split: b</code>

Backtracking again —and again

Again, at this point `k = b.length`, the frame for the call is popped and the value of `s1 == s2` is returned—it is again false. Backtracking occurs in order to attempt a different choice. But this time, it is the second call on `split` that returned false, so the frame for the call is popped and the value of `s1 == s2` is returned—again it is false. More backtracking.

Eventually, the call `split(b, 1, 1, 0)` will return false.

false

<code>split: 0, 0, 0</code>
<code>split: b</code>

This will cause the second call on `split` to be executed, with the first 1 being put in the second bag. Now, a lot of calls and backtracking will take place before the call `split(b, 0, 0, 0)` returns false.

Worst-case number of calls on split

We modified function `split` to also calculate how many calls on `split` are made. See our revised method in the demo code that accompanies entry `backtracking` in JavaHyperText. When the values in array `b` of size `n` cannot be split into two bags so that their sums are the same, $2^{(n+1)}$ calls are made! Thus, `split` takes time $O(2^n)$ in the worst case.

We could have predicted this. Each value can be placed in bag 1 or bag 2. There are 2^n sequences of 1's and 2's of length `n`, and each has to be tested by `split`.