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Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric, with eigenvalue decomposition $Q \Lambda Q^T$ where the eigenvalues are sorted in descending order of magnitude and $|\lambda_1| > |\lambda_2|$. If we write the eigenvector basis as $Q = \begin{bmatrix} q_1 & Q_2 \end{bmatrix}$, The cosine and sine of the acute angle between q_1 and a unit vector v are $\cos \angle(q_1, v) = |q_1^T v|$ and $\sin \angle(q_1, v) = \sqrt{1 - |q_1^T v|^2} = \|Q_2^T v\|$. Using these definitions, argue that when v_k is the kth step of power iteration,

$$\tan \angle(q_1, v_k) \leq |\lambda_2/\lambda_1|^k \tan \angle(q_1, v_0).$$