HW for 2020-02-14

(due: 2020-02-21)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Describing definiteness Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric and positive definite. Give *brief* proofs of each of the following facts

- A is invertible.
- A^{-1} is positive definite.
- Any principal submatrix is positive definite.
- Any Schur complement is positive definite.

2: Beyond Cholesky Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric and positive definite and $B \in \mathbb{R}^{n \times k}$ has full column rank (with $k < n$). Argue for the existence of the factorization

$$\begin{bmatrix} A & B \\ B^T & 0 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}^T$$

and give a short code (using all the standard built-ins) to compute this factorization in the language of your choice.

3: Recover the map Suppose we are given N pairs $(x^{(i)}, y^{(i)}) \in \mathbb{R}^n \times \mathbb{R}^n$. Write a short code (in any language) to solve

$$\text{minimize } \sum_{i=1}^{N} \|y^{(i)} - Ax^{(i)}\|^2 \text{ over } A \in \mathbb{R}^{n \times n}$$

You may use built-in functions for solving least squares. Illustrate your code on a sample set of points.