HW for 2020-02-07

(due: 2020-02-14)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Growing a system Suppose $PA = LU$ is given. Write an $O(n^2)$ time algorithm to extend the factorization to an LU factorization of

$$M = \begin{bmatrix} A & b \\ c^T & d \end{bmatrix}.$$

Write your code as a function that takes P, L, U, b, c, and d as inputs, and returns extended matrices $\bar{P}$, $\bar{L}$, and $\bar{U}$. You should verify that your code is correct for random choices of A, b, c, and d by checking the backward error, i.e. $\|\bar{P}M - \bar{L}\bar{U}\|_F / \|M\|_F$ should be small.

2: Shrinking back Suppose $PA = LU$ is given. Consider the system $Ax = b + re_i$ where $x_j = 0$; that is, we allow the ith equation not to be satisfied (by an unknown amount r), but enforce that $x_j = 0$. Express this new problem in terms of a bordered system, and use block elimination to give an $O(n^2)$ code to compute x and r.

3: Iterative refinement and bordering The straightforward algorithm in problem 2 runs into stability problems when A is nearly rank deficient, even if the modified system is well-conditioned. Illustrate the problem with the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1+\delta \end{bmatrix}$$

for $\delta = 10^{-12}$. Show experimentally that a step of iterative refinement fixes the issue.