HW for 2020-01-24

(due: 2020-01-31)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Placing parens Suppose $A, B \in \mathbb{R}^{n \times n}$ are square matrices, $D = \operatorname{diag}(d) \in \mathbb{R}^{n \times n}$ is a diagonal matrix, and $u, v \in \mathbb{R}^n$ are vectors. Write short fragments of Julia, MATLAB, or Python to evaluate each of the following expressions in the inidicated complexity

- tr(DAD) in $O(n)$
- $v^T(I + uu^T)v$ in $O(n)$
- $u^T A^2 v$ in $O(n^2)$

2: Making matrices In terms of the power basis, write

- $A \in \mathbb{R}^{5 \times 4}$ corresponding to multiplication of a polynomial of degree at most 3 by x.
- $B \in \mathbb{R}^{5 \times 5}$ corresponding to differentiation of quartics.
- A symmetric $M \in \mathbb{R}^{3 \times 3}$ representing the quadratic form $p(x) \mapsto \int_0^1 p(x)^2 \, dx$ for polynomials $p(x)$ of maximum degree 2; that is,

$$\int_0^1 p(x)^2 \, dx = c^T M c$$

  where $c \in \mathbb{R}^3$ is the vector of coefficients for the polynomial.

3: Low rank limbo SSuppose $u, v \in \mathbb{R}^n$ and let $L = uv^*$. Show that

- $\|L\|_1 = \|u\|_1 \|v\|_\infty$
- $\|L\|_\infty = \|u\|_\infty \|v\|_1$
- $\|L\|_F = \|u\|_2 \|v\|_2$