# Midterm

You should not consult with anyone inside or outside the class, except for the course staff. You may use textbooks or online sources *with citation.*

**1: Harder, faster, better, stronger** Rewrite the following MATLAB code fragments to perform an equivalent computation with better efficiency. Assume a Cholesky factor $A = R^T R$ is precomputed. In each case, also state the complexity of the revised operation:

```
x1 = diag(d) * z; % d and z are vectors of length n
x2 = trace(u*v'); % u and v are vectors of length n
B = inv(A); x3 = B(1,1); % We only care about x3, not B
x4 = (eye(n) + u*v') \ z;
```

**2: Norm!** Let $M$ be an invertible matrix, and define

$$\|v\|_* = \|Mv\|_\infty.$$

Argue *briefly* that this is indeed a vector space norm, and write a MATLAB code to compute the associated operator norm.

```
function [normA] = mt_norm(M, A)
```

**3: Floating point fandango** For $x > 1$, the equation $x = \cosh(y)$ can be solved as

$$y = -\log\left(x - \sqrt{x^2 - 1}\right).$$

Describe *briefly* why this formula fails to achieve good accuracy when $x = 10^8$ (for example). Write an alternate code that retains good accuracy for such large $x$ values:

```
function [y] = my_acosh(z)
```

**4: Bordered bonanza** Suppose `x = solveA(b)` is a black-box routine that solves the (well-conditioned) linear system of equations $Ax = b$. Construct a new routine that satisfies all but the $k$th row of $Ax = b$, subject to the constraint that $x_\ell = 0$.

```
function [x] = solveA_constrain(solveA, b, k, l)
```

**5: Almost rank one** Given $A \in \mathbb{R}^{n \times m}$ and $b \in \mathbb{R}^n$, find $u \in \mathbb{R}^m$ so that $\|A - bu^T\|_F^2$ is as small as possible. Please use the fact that MATLAB backslash solves a least squares problem if the matrix involved is rectangular.

```
function [u] = approx_rank1(A, b)
```