HW 4

1: Crossing cubics A *Bezier curve of degree* n is the curve traced out by

$$f(t) = \sum_{i=0}^{n} p_i B_i^n(t), \quad t \in [0,1]$$

where the points $p_i \in \mathbb{R}^2$ are *control points* and the functions $B_i^n(t)$ are the *Bernstein polynomials*

$$B_i^n(t) = C_i^n (1-t)^{n-i} t^i, \quad C_i^n = \frac{n!}{i!(n-i)!}.$$

A common type of Bezier curve in computer graphics is the *cubic Bezier curve* defined by four control points. Complete the following function to compute the intersection of two such curves

```
function [x] = bezier_intersect(pf, pg)
%
% Attempt to compute the intersection of the cubic Bezier curves
% defined by the columns of pf and pg (each of dimension 2-by-4).
% Assume a unique intersection.
```

Illustrate with an example that your solution works correctly.

2: Funky fixed point Argue that the iteration

$$\begin{aligned} 3x_{k+1} + 2y_{k+1} &= \cos(x_k) \\ 2x_{k+1} + 4y_{k+1} &= \cos(y_k) \end{aligned}$$

converges to a unique fixed point (x_*, y_*), regardless of the initial point, and that $\|e_{k+1}\|_2 < 0.7\|e_k\|_2$, where $e_k = (x_k - x_*, y_k - y_*)$. Starting from the point $(1,1)$, draw a semi-logarithmic plot of the error versus k to illustrate the convergence.