HW 1

The first two problems should be plausible given what you know as of Jan 27. Ideally, the material presented on Monday, Jan 30 will allow you to do the other two problems. Don't be shy about asking for help in office hours or on Piazza!

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Placing parens Suppose $A, B \in \mathbb{R}^{n \times n}$ are square matrices, $D = \operatorname{diag}(d) \in \mathbb{R}^{n \times n}$ is a diagonal matrix, and $u, v \in \mathbb{R}^n$ are vectors. Write short fragments of MATLAB or Julia to evaluate them as efficiently as possible, and give the complexity in terms of n:

1. $\operatorname{tr}(DAD)$
2. $v^T(I + uu^T)v$
3. $u^T A^2 v$

2: Pretty pictures Reproduce the matrix-matrix product timing plots from lecture 1 for your own machine. Note that the MATLAB code to run these experiments is in the `lec/code` subdirectory of the course GitHub page, which is linked from the home page. If you would like to translate this into Julia, it should be quite straightforward!

3: Making matrices In terms of the power basis, write matrices corresponding to the following linear maps:

1. $A \in \mathbb{R}^{5 \times 4}$ corresponding to multiplication of cubics by x.
2. $B \in \mathbb{R}^{5 \times 5}$ corresponding to differentiation of quartics.

4: Crafty cosines Suppose $\|\cdot\|$ is an inner product norm in some space, and you are given

$$a = \|u\|, \quad b = \|v\|, \quad c = \|u + v\|.$$

Write a formula in terms of a, b, c for the cosine of the angle between vectors u and v.