CS 4110

Programming Languages & Logics

Lecture 37
Typed Assembly Language

28 November 2012

Schedule

Monday

• Typed Assembly Language

Today

- Polymorphism
- Stack Types

Friday

- Compilation
- Course Review

TAL-0 Review

- Syntax
- Semantics
- Type System
 - \blacktriangleright Ψ ; $\Gamma \vdash v : \tau$
 - ▶ $\Psi \vdash i : \Gamma \rightarrow \Gamma'$
 - au $au \leq au'$

- ► *H* : Ψ
- **▶** ⊢ R : **Γ**
- ightharpoonup \vdash (H, R, B)

TAL-0 Review

- Syntax
- Semantics
- Type System

Theorem (Type Safety)

If $\vdash \Sigma$ and $\Sigma \mapsto^* \Sigma'$, then Σ' is not stuck.

Lemma (Progress and Preservation)

- If $\vdash \Sigma_1$ then there exists a Σ_2 such that $\Sigma_1 \mapsto \Sigma_2$
- If $\vdash \Sigma_1$ and $\Sigma_1 \mapsto \Sigma_2$ then $\vdash \Sigma_2$

TAL-1: Polymorphism

Syntax

- Add type variables α and universal types $\forall \alpha. \tau$
- Allow code label types to be polymorphic $\forall \alpha, \beta. \{r_1 : \alpha, r_2 : \beta, r_3 : \{r_1 : \beta, r_2 : \alpha\} \rightarrow \{\}\} \rightarrow \{\}$
- Add type application $v[\tau]$
- Write $v[\tau_1, \ldots, \tau_k]$ for $v[\tau_1] \cdots [\tau_k]$

Polymorphism Example

```
swap: \forall \alpha, \beta . \{r_1 : \alpha, r_2 : \beta, r_{31} : \{r_1 : \beta, r_2 : \alpha\} \rightarrow \{\}\} \rightarrow \{\}
      mov r_3, r_1
      mov r_1, r_2
      mov r_2, r_3
      imp r_{31}
swap\_ints: \{r_1: int, r_2: int, r_{31}: \{r_1: int, r_2: int\} \rightarrow \{\}\} \rightarrow \{\}
      imp swap [int, int]
swap\_int\_and\_label: \{r_1: int, r_2: \{r_2: int \rightarrow \{\}\}\}
      mov r_{31}L
      imp swap [int, \{r_2: int\} \rightarrow \{\}]
L: \{r_1: \{r_2: \text{int}\} \to \{\}, r_2: \text{int}\} \to \{\}
      jmp r_1
```

Callee-Saves Registers

Common Strategy

- When calling a function...
- Save the contents of some registers on the stack...
- Allow the callee to save (and restore) other designated registers...
- If the callee does not use all registers, the cost of saving and restoring is not incurred...

Correctness Critereon

Callee must return the callee-saves registers to the caller with the same values as when the function was invoked.

Callee-Saves Example

```
callee: \forall \alpha. \{r_1 : \text{int}, r_5 : \alpha, r_{31} : \{r_1 : \text{int}, r_5 : \alpha\} \rightarrow \{\}\} \rightarrow \{\}
      mov r_4, r_5
                                   % Save r_5
                                   % Use r_5 for other work
     mov r_5,7
     add r_1, r_1, r_5
                                   % Restore r_5
     mov r_5, r_4
     imp r_{31}
caller: \{\} \rightarrow \{\}
     mov r_5,255
     mov r_1,5
     mov r_{31},L
     jmp callee[int]
L: \{r_1 : \text{int}, r_5 : \text{int}\} \to \{\}
     mul r_3, r_1, r_5
```

Callee-Saves Bug

```
callee: \forall \alpha. \{r_1 : \text{int}, r_5 : \alpha, r_{31} : \{r_1 : \text{int}, r_5 : \alpha\} \rightarrow \{\}\} \rightarrow \{\}
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                                   % Use r_5 for other work
     mov r_5,7
     add r_1, r_1, r_5
                                   % Restore r_5
     mov r_5, r_4
                                   % Error! r_5:int
     imp r_{31}
caller: \{\} \rightarrow \{\}
     mov r_5,255
     mov r_1,5
     mov r_{31},L
     jmp callee[int]
L: \{r_1 : \text{int}, r_5 : \text{int}\} \to \{\}
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```

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- This follows directly from callee's polymorphic type!

$$\forall \alpha. \begin{cases} r_1 : \text{int,} \\ r_5 : \alpha, \\ r_{31} : \{r_1 : \text{int,} r_5 : \alpha\} \to \{\} \end{cases} \to \{\}$$

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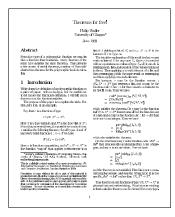
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- Types can also be used to constrain the behavior of functions

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- Types can also be used to constrain the behavior of functions



Paper: P. Wadler. "Theorems for Free!" In *FPCA*, pp. 347–359. September 1989.

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• Heaps H now map labels to type-labeled blocks:

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- Type variables may appear free in both Γ and B
- Control-flow operations substitute types $(H, R, \text{jmp } v[\tau_1, \dots, \tau_k]) \mapsto (H, R, B[\tau_1/\alpha_1, \dots, \tau_k/\alpha_k])$ where R(v) = L and $H(L) = \forall \alpha_1, \dots, \alpha_k$. $\Gamma \rightarrow \{\}$. B

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Typing Polymormphism

$$\Psi; \Delta; \Gamma \vdash v : \tau$$

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Type application

$$\frac{\Psi; \Delta; \Gamma \vdash \nu : \forall \alpha_1, \dots, \alpha_k. \Gamma' \to \{\} \qquad \Delta \vdash \tau}{\Psi; \Delta; \Gamma \vdash \nu [\tau] : (\forall \alpha_2, \dots, \alpha_k. \Gamma' \to \{\})[\tau/\alpha]}$$

TAL-2: Stack Types

Run-Time Stack

Almost every compiler uses a run-time stack

- What is a stack?
- A consecutive sequence of memory addresses with one end designated as the *top* of the stack
- Values are stored on the top of the stack and retrieved later
- The compiler may grow or shrink the stack as needed

Stack uses

- Local variables
- Spilled registers
- Return addresses

Stack Syntax

Machine states:

$$M ::= (H, R, S, B)$$

Stacks:

$$S ::= [] | v :: S$$

• Instructions:

$$i ::= \cdots \mid \text{salloc } n \mid \text{sfree } n \mid \text{sld } r_d, n \mid \text{sst } v, n$$

- Errors:
 - Free too many values
 - ► Read too deep in the stack
 - Write too deep in the stack

Stack Instructions

The new stack instructions can be easily encoded:

- A designated register *sp* points to the top of the stack
- salloc *n* subtracts *n* from *sp* (i.e., sub *sp*, *sp*, *n*)
- sfree *n* adds *n* to *sp* (i.e., add *sp*, *sp*, *n*)
- sld r_d , n reads a value at offset n relative to sp (i.e., $ld r_d$, sp(n))
- sst v, n writes a value at offset n relative to sp (i.e., st sp(n), v)

CISC-like stack instructions can also be encoded:

- push v is salloc 1; sst v, 1
- pop r_d is sld r_d , 1; sfree 1

Example: Factorial

```
fact(n) =
if n \le 0 then 1
else n \times fact(n - 1)
```

Example: Factorial

```
fact:
       bgt r_1,L1
                          % if n > 0, goto L1
        mov r_1,1
                          % if n < 0, return
       jmp r_{31}
L1:
       salloc 2
                          % allocate space for frame
                          % save return address
        sst r_{31},1
        sst r_1,2
                          % save n
       sub r_1, r_1, 1
                          % n := n - 1
                          % set return address
        mov r_{31},L
       jmp fact
                          % recursive call
       sld r_2,2
                          % restore n
       sld r_{31}, 1
                          % restore return address.
        sfree 2
                          % free space for frame
                          % result := n \times fact(n-1)
        \operatorname{mul} r_1, r_1, r_2
                          % return
       jmp r_{31}
```

$$(H, R, S, \text{salloc } n; B) \mapsto (H, R, ? :: \cdots :: ? :: S, B)$$

$$(H, R, S, salloc n; B) \mapsto (H, R, ? :: \cdots :: ? :: S, B)$$

$$(H, R, v_1 :: \cdots :: v_n :: S, \text{ sfree } n; B) \mapsto (H, R, S, B)$$

$$\overline{(H,R,S,\text{salloc }n;B) \mapsto (H,R,?::\cdots::?::S,B)}$$

$$\overline{(H,R,v_1::\cdots::v_n::S,\text{sfree }n;B) \mapsto (H,R,S,B)}$$

$$S = v_1::\cdots::v_n::S'$$

$$\overline{(H,R,S,\text{sld }r_d,n;B) \mapsto (H,R[r_d:=v_n],S,B)}$$

$$\overline{(H,R,S,\text{salloc }n;B)} \mapsto (H,R,?::\cdots::?::S,B)$$

$$\overline{(H,R,v_1::\cdots::v_n::S,\text{sfree }n;B)} \mapsto (H,R,S,B)$$

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$$S = v_1::\cdots::v_n::S'$$

$$\overline{(H,R,S,\text{sst }v,n;B)} \mapsto (H,R,v_1::\cdots::R(r_s)::S',B)$$

Type System

Syntax

$$\sigma ::= [] \mid \tau :: \sigma \mid \rho$$

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- ρ is a stack type variable
- Register file types contain a special variable sp

```
\{sp : int :: int :: [], r_1 : int, ...\}
```

Syntax

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- Register file types contain a special variable sp

$$\{sp : int :: int :: [], r_1 : int, ... \}$$

Code label types can be polymorphic over stack types

$$\forall \rho. \{ sp : int :: \rho, r_1 : int \} \rightarrow \{ \}$$

Syntax

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Code label types can be polymorphic over stack types

$$\forall \rho. \{ sp : \text{int} :: \rho, r_1 : \text{int} \} \rightarrow \{ \}$$

Junk values "?" have junk types "?"

$$\boxed{\Psi;\Delta\vdash i:\Gamma_1\to\Gamma_2}$$

$$\Psi$$
; $\Delta \vdash i : \Gamma_1 \rightarrow \Gamma_2$

Stack allocation

$$\frac{\Gamma(sp) = \sigma}{\Psi; \Delta \vdash \mathsf{salloc}\, n : \Gamma \to \Gamma[sp :=? :: \cdots ::? :: \sigma]}$$

$$\Psi$$
; $\Delta \vdash i : \Gamma_1 \rightarrow \Gamma_2$

Stack allocation

$$\frac{\Gamma(sp) = \sigma}{\Psi; \Delta \vdash salloc \ n : \Gamma \rightarrow \Gamma[sp :=? :: \cdots ::? :: \sigma]}$$

Stack free

$$\frac{\Gamma(sp) = \tau_1 :: \cdots :: \tau_n :: \sigma}{\Psi; \Delta \vdash \text{sfree } n : \Gamma \to \Gamma[sp := \sigma]}$$

$$|\Psi; \Delta \vdash i : \Gamma_1 \rightarrow \Gamma_2|$$

Stack load

$$\frac{\Gamma(sp) = \tau_1 :: \cdots :: \tau_n :: \sigma}{\Psi; \Delta \vdash \mathsf{sld} \ r_d, n : \Gamma \to \Gamma[r_d := \tau_n]}$$

$$\Psi$$
; $\Delta \vdash i : \Gamma_1 \rightarrow \Gamma_2$

Stack load

$$\frac{\Gamma(sp) = \tau_1 :: \cdots :: \tau_n :: \sigma}{\Psi; \Delta \vdash \mathsf{sld} \, r_d, n : \Gamma \to \Gamma[r_d := \tau_n]}$$

Stack store

$$\frac{\Psi; \Delta; \Gamma \vdash v : \tau \qquad \Gamma(sp) = \tau_1 :: \cdots :: \tau_n :: \sigma}{\Psi; \Delta \vdash sst v, n : \Gamma \rightarrow \Gamma[sp := \tau_1 :: \cdots :: \tau :: \sigma]}$$

Example: Factorial Bug

```
fact: \forall \rho. {sp : \rho, r_1 : int, r_{31} : {r_1 : int, sp : \rho} → {}} \rightarrow {}
      bgt r_1,L1[\rho]
      mov r_1,1
      jmp r_{31}
L1: \forall \rho. {sp: \rho, r₁: int, r₃₁: {r₁: int, sp: \rho} → {}} → {}
      salloc 2
      sst r_{31},1
      sst r_1,2
      sub r_1, r_1, 1
      mov r_{31},L[\rho]
      imp fact
L: \forall \rho. \{ sp : \{ r_1 : int, sp : \rho \} \rightarrow \{ \} :: int :: \rho, r_1 : int \} \rightarrow \{ \}
      sld r_2,2
      sld r_{31}, 1
      sfree 2
      \text{mul } r_1, r_1, r_2
                                % Error! sp: \{r_1: int, sp: \rho\} \rightarrow \{\}:: int:: \rho
      jmp r_{31}
```

Example: Callee Bug

```
caller: \forall \rho. \{sp : \tau_{code} :: \rho\} \rightarrow \{\}
       salloc 1
       mov r_1, 17
       sst r_1,1
       mov r_{31},L[\rho]
      imp\ callee[\tau_{code} :: \rho]
callee: \forall \rho. {sp : int :: \rho, r_{31} : \{sp : \rho, r_1 : int\} \rightarrow \{\}\} \rightarrow \{\}
       sld r_1,1
       add r_1, r_1, r_1
       sst r_1,2
                                                  % Error!
       sfree 1
      jmp r_{31}
L: \forall \rho. \{ sp : \tau_{code} :: \rho, r_1 : int \} \rightarrow \{ \}
```

Type Safety

- Type safety ensures we don't get stuck
- With a few additional features, can handle exceptions
- Paper: G. Morrisett, K. Crary, N. Glew, and D. Walker.
 "Stack-based Typed Assembly Language." In *JFP*. 12(1):43–88.
 January 2002.

J. Functional Programming 12 (1): 43–65. January 2002. © 2002 Cambridge University Press

Stack-based typed assembly language

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Abstract

This paper presents STAL a variety of Typed Anomably Language with constructs and pages to support a baseline from of such allocates. As who whose statisticately-paged so-level design to support a baseline of the such as Language STALA. As appear extensive as a consent in an excess rule in the base by variety and the such as Language STALA. Such appears that the such as the such as Language STALA as the such a

Cansule Review

The shifty to type-their baseline describble code glops an important rule in counting and controlled and instant code in a source controlled and in the popular subset does, and more provided kernel extension. Bytecode verification in has in a well-known example of type-checking excentible code, but a specific use to a specific their high-level virus machine interestion set. Typed Ausembly Language (TAL), introduced by Marrient or al. in 1999, extends this supposals is much lower-off excentable code in provides a flood byer system for a language similar to the machine code of contemporary processors. However, one limitation of TAL is that in applice only to code compeling incintation-possing right, task in

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- Polymorphism is critical
- Can encode many calling conventions:
 - Arguments on stack or in registers?
 - Results on stack or in registers?
 - Return address: caller pops? callee pops?
 - Registers: caller saves? callee saves?

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Moral: orthogonal combination of type system constructs makes it easy to scale language features