Problems for October 31

The key to counting things accurately is to organize how you count so you don't forget to count some things.

An *anagram* of a word is a way to put its letters into a new order, even if that new order does not make a word. For example, think about the word "CAT". One anagram is "ACT", which is a word, but also "TCA" is an anagram. Anagrams are an example of a more general idea called a *permutation*: a way of putting things into a different order.

1. List all six anagrams of "CAT" (which include "CAT" itself).

   Answer:

   CAT, CTA, TAC, TCA, ACT, ATC

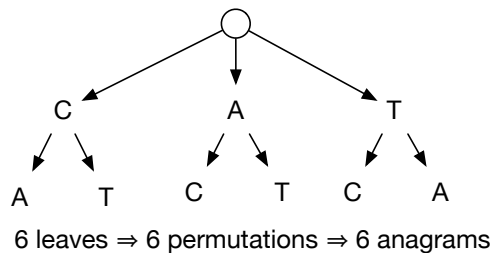2. How many anagrams does "DOG" have?

   Answer:

   6

3. How many anagrams does "ALL" have?

   Answer:

   3

If a word has all different letters then it is easy to figure out how many anagrams it has. For example, think about the word "CAT", Without listing its anagrams, we can see that there are 6. Consider picking the letters one by one. One of the 3 letters could be the first letter. Once we've picked it, any one of the remaining 2 letters could be the second letter. And finally there is only one letter we can choose. The possible ways we can choose letters can be drawn as a tree called a *decision tree*:

6 leaves ⇒ 6 permutations ⇒ 6 anagrams

So the total number of anagrams is $3 \times 2 \times 1 = 6$.

4. How many anagrams does the word "MATH" have? Don't list all of them!

   Answer:

   24

5. How many anagrams does "GHOST" have?

   Answer:

   5×4×3×2×1 = 120

6. How many anagrams does "HELLO" have? Notice that just like with "ALL", if you swap the two L's, you don't get a new word!
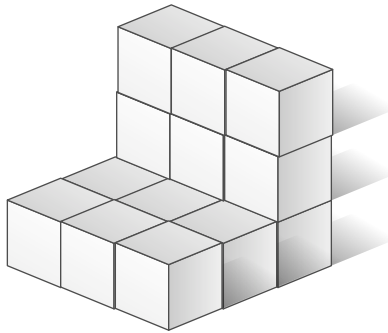
   Answer:

   120/2 = 60

7. How many anagrams does your name have? How about "HALLOWEEN"?

   Answer:

   It depends!

8. The following three-dimensional shape is made out of cubes. You can see all the cubes but two. How many cubes are there total?

   Answer:

   15

9. Suppose it takes one can of paint to paint a single cube on all sides. How many cans will it take to paint the whole shape, including the bottom and the back sides? When two cubes are attached to each other, we don't paint the sides that touch.
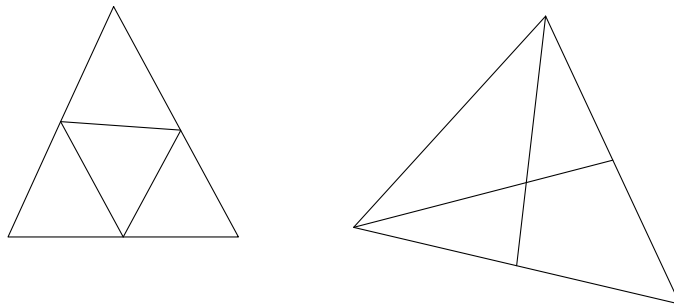
   Answer:
   Think about looking at the cube from each of the 6 directions. In all but two directions, we see 9 squares; in the other two directions we see an L shape covering 5 squares. So we have 9+9+9+9+5+5 = 46 sides. Therefore, it will take 8 cans of paint, though some paint will be left over at the end.

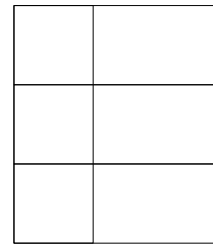10. How many triangles are there in the figure on the left?

    Answer:

    5

11. How many triangles are there in the figure on the right?

    Answer:

    8

12. How many rectangles are there in this figure?

    Answer:

    $6 \times 3 = 18$