Problems for October 24

When we multiply a number by itself, we get a *square*. The first few squares are 0, 1, 4, 9, 16. Instead of writing 4×4, there is a special way to write it: 4^2. Below is a list of the first 20 squares. Can you memorize all of them? Pick a partner and test each other. Make sure you know the first 10 squares (up to 100) first!

Here is a cool fact about squares: if you multiply two numbers whose difference is 1, the result is one less than the square of the number between them. For example, $7 \times 9 = 63 = 8^2 - 1$.

Here is another cool fact: if you add all the odd numbers up to some number, the result is always a square: 1+3 = 4, 1+3+5 = 9, and so on.

x	x^2
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

1. What is 30^2?

2. Two consecutive squares differ by 17. What is the smaller of the two numbers being squared?

3. Two consecutive squares differ by 33. What is the smaller of the two numbers squared?

4. Two consecutive squares differ by 43. What is the smaller of the two squares?

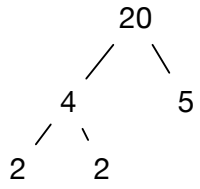
5. How many squares are there between 401 and 899?

6. What is $8 \times 201 - 8 \times 199$? Can you figure it out without doing big multiplication problems?

7. What is $17 \times 321 - 17 \times 302$? Can you use the table of squares to help?

A factorization tree for a number is a tree that has the number at the root (at the top), where each number that is not prime has branches that multiply to make that number and that are greater than 1. For example, here is a factorization tree for 20. The leaves must be prime numbers.

Once we have made the factorization tree, we can sort all the leaves and make the *prime factorization* of the number. For example, the prime factorization of 20 is $2 \times 2 \times 5$.



8. Draw factorization trees for the following numbers: 8, 12, 30, 36, 120, 225, and write out the prime factorization of each number.

9. What are all the factors of 24 greater than 1, and what are their prime factorizations? Do you see any pattern?