

## Magic Squares and Pascal's Triangle

A *magic square* is a square grid of some size  $n$ , containing all the whole numbers between 1 and  $n^2$ . The numbers on every row, column, and the two diagonals always add up to the same number.

Here is a magic square of size 3:

8	1	6
3	5	7
4	9	2

Every row, column, and diagonal adds up to 15. For a square of size  $n$ , the rows, columns, and diagonals always add to  $n \times (n^2 + 1)/2$ .

1. Can you complete the following magic square?

2	9	
7		

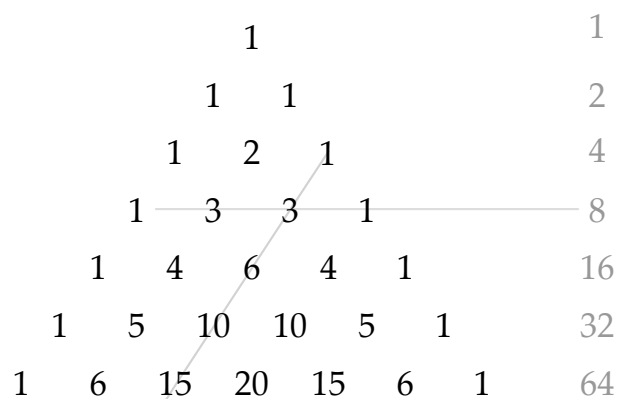
2. How about this size-4 magic square? (What should the rows and columns add up to?)

16	2	3	
5	11	10	
9			

3. You can make an odd-sized magic square of *any size* by following a simple procedure. Place the number 1 in the top center square. Then count upward while going up to the right. But if you ever fall off the sides of the square, you “wrap around” to the other side. If the square you were going to write into is already occupied by a number, you move down a square instead of up to the right. Here is a  $5 \times 5$  magic square in which this process has been started. Can you finish it?

		1		
	5			
4	6			
10				3
			2	

Pascal's triangle is a triangle of numbers in which every number is the sum of the two numbers directly above it (or is 1 if it is on the edge):



triangular  
numbers

each row adds  
to a power of 2

If you color the entries in Pascal's triangle according to their remainder when dividing by some number, cool patterns show up. You can try coloring all the odd numbers one color and all the even ones another color.

For something more fancy, color all the numbers that are multiples of 3 one color and all the numbers that are one greater than a multiple of 3 a second color.