Next week: Fall Noetic contest!

Cayuga Heights 5th Grade Math Club
Problems for November 6, 2015

Permutations, combinations, and Pascal's triangle

1. How many different ways are there to order the letters in the word `MATH`?

   > The number of *permutations* of a sequence of distinct objects is the *factorial* of the number n of objects: $n! = 1 \times 2 \times 3 \times \ldots \times n$.

   Followup: how about `THUMB`?

2. How many different ways are there to order the letters in the word `AROMA`?

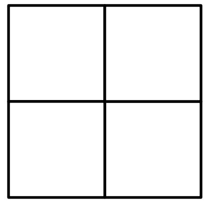
3. How many different ways are there to order the letters in the word $\mathtt{A}_1\mathtt{GA}_2\mathtt{VA}_3$, where $\mathtt{A}_1$, $\mathtt{A}_2$, and $\mathtt{A}_3$ are all different letters? What if we erase the subscripts (1,2,3) so all three `A`'s are the same?

   Followup: how many ways are there to order the letters in your own name?

4. How many different ways are there to order the letters in the word `SSEE`?

   > We write $\binom{5}{2}$ (read as "5 choose 2") to mean the number of ways to choose 2 items from among 5. This is the same thing as the number of ways to order the letters `AABBB`, where an `A` means that an item was chosen and a `B` means that the item was not chosen. The formula for "choose" is as follows:
   >
   > $$\binom{n}{m} = \frac{n!}{m! \times (n-m)!}$$
   >
   > The top of the fraction gives us the total number of permutations of n items. But since there are m A's and $n - m$ B's, we need to divide by the factorials of m and $n - m$.

5. Theo is two blocks northwest of where he wants to go. Assuming he only travels South or East to get to his destination, how many different ways are there for him to get there?



Followup: what if he has to go three blocks east and three blocks south?

6. If a family has 4 children, then assuming children are equally likely to be boys or girls, what is the probability that there are two boys and two girls? (that is, what fraction of the time should we expect this to happen?)

Pascal's triangle is a triangle of numbers in which every number is the sum of the two numbers directly above it (or is 1 if it is on the edge):

1	1
1 1	2
1 2 1	4
1 3 3 1	8
1 4 6 4 1	16
1 5 10 10 5 1	32
1 6 15 20 15 6 1	64

triangular numbers

each row adds to a power of 2

The entries of Pascal's triangle tells us the number of ways to choose items. For example, in row 4 the middle element tells us: $\binom{4}{2}$, which is also the number of permutations of `AABB`, which is also the number of ways to go 2 blocks south and 2 blocks east.

Notice that when n is a prime number, all of the numbers in row n, except 1, are divisible by n. That is because they all have the form $\frac{n!}{m! \times (n-m)!}$. If n is prime, the factor of n in the top factorial cannot be canceled out by the smaller factorials on the bottom.