# Right triangles and the Pythagorean Theorem

Right triangles have some special properties that can be very helpful when solving math problems. A right triangle is a triangle with a single right angle. The side opposite the right angle is called the *hypotenuse* and the other two sides are called the *legs* of the triangle. The hypotenuse of a right triangle is always the longest side of the triangle, because the angles opposite the legs of the triangle are always *acute* angles (smaller than right angles).

The area of a right triangle is computed using the same formula as the area for any triangle; one-half of the base of the triangle times its height. It's particularly convenient for right triangles because the two legs are the base and the height.

1. The sail of Charlotte's boat is a right triangle that is 15 feet tall and 8 feet wide along the bottom. What is the area of the sail?

2. When shining in the direction of a wall 12 feet away, Matthew's movie projector illuminates a triangular region of the floor. If the illuminated region is 8 feet wide at the widest point where it hits the wall, what is the area of the floor that is illuminated?

Remember that the angles of a triangle always add up to 180 degrees. Since a right angle is 90 degrees, that means the other two angles add up to 90 degrees. They are said to be *complementary* angles. For example, that means we can fit a right triangle together with a copy of itself to make a rectangle.

3. Four copies of the same right triangle are glued together as shown. How big is the blue angle in degrees?
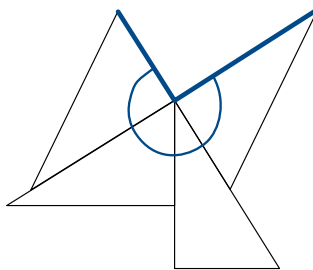
4. Two copies of the same right triangle are attached so one leg of one is attached to the other leg of the second, as shown. What is the size of the indicated angle, in degrees?
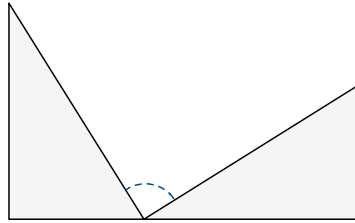
Triangles are very useful for area problems because we can often break up more complex shapes into triangles and then add and subtract areas to get the solution.

5. The large square shown contains smaller squares A, B, C whose areas are 4, 9, and 4 sq cm respectively. What is the area of the shaded region?
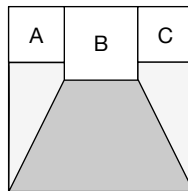
Suppose we have a right triangle whose legs have lengths 3 and 4, and we attach a square A onto its hypotenuse as shown. What is the area of the triangle? What is the area of the square?
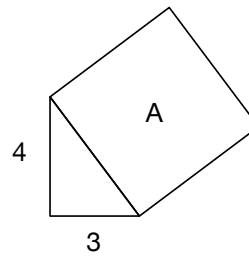
It turns out there is a famous formula to figure this out. Let's just think about this particular triangle. We can attach three more copies of it to itself to make a big square, as shown.
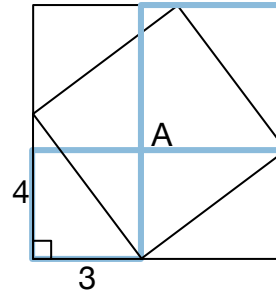
We can see that the area of the big square is $7^2 = 49$, and then subtract off the area of four triangles (24) to get 25. An even more useful way to see this is that we can break up the big square as shown into two squares and two rectangles. The areas of the two squares are $3^2 = 9$ and $4^2 = 16$. The area of the two rectangles is the same as four triangles. Since the two new squares plus four triangles have the same area as square A plus four triangles, the area of square A is the same as the total area of the two squares. That is, $3^2 + 4^2 = 25$.

Nothing about this argument depended on the numbers 3 and 4. So for *any* right triangle with legs of length a and b, the area of a square built on the hypotenuse is $a^2 + b^2$. If we call the hypotenuse c, the area is c^2. So for any right triangle, $a^2 + b^2 = c^2$. This fact is called the Pythagorean Theorem, named after the Greek mathematician Pythagoras.

6. If a right triangle has leg lengths 6 and 8, what is the length of the hypotenuse?

7. If a right triangle has a hypotenuse of length 13 and one leg of length 5, what is the length of the other leg?

8. For an isoceles right triangle whose legs are length 10, what is the length of the hypotenuse to the nearest integer?

Any right triangle whose sides all have integral length is called a *Pythagorean triangle*. A 3–4–5 triangle is the smallest Pythagorean triangle. We call (3,4,5) a Pythagorean triple. Other famous Pythagorean triples are (5, 12, 13) and (8, 15, 17), but there are an infinite number of them!