One's digits and remainders (modular arithmetic)

When we add, subtract, or multiply two numbers, the one's digit of the result only depends on the one's digit of the operands. For example, we don't need to figure out what 2314×1117 is to know that the one's digit must be 8.

1. What is the one's digit of the result of 2883×799912?

   Answer:

   $3 \times 2 = \boxed{6}$

2. What is the one's digit of the result of 3487623×11117?

   Answer:

   $3 \times 7 = 21$, *so the one's digit must be* $\boxed{1}$.

3. What is the one's digit of the result of $(21783 + 9838) \times 8917$?

   Answer:

   $3 + 8 = 11$, *and* $1 \times 7 = \boxed{7}$.

4. If the remainder when two numbers are divided by 10 is 7, what is the remainder of their product when divided by 10?

   Answer:

   $7 \times 7 = \boxed{9}$

There is nothing special about the number 10. The same thing works when we write a number in any number base. The one's digit of a number in some base N is the same as the remainder when we divide by N. So to know what the one's digits (the remainder) of some computation is, we only need to know the remainder for all the numbers involved in the computation!

5. What does the base-8 number 41 represent in base 10? Show how to multiply this number by itself *in base 8*. Notice that the one's digit must be 1!

   Answer:

$$\begin{array}{rrrr} & & 4 & 1 \\ \times & & 4 & 1 \\ \hline & & 4 & 1 \\ 2 & 0 & 4 & \\ \hline 2 & 1 & 0 & 1 \end{array}$$

6. Consider the base-2 numbers 100101_2 and 10101_2. What is the one's digit of their product? Is the product even or odd?

   Answer:

   $1 \times 1 = 1$, *so the product must be odd.*

7. Consider the base-8 numbers 45_8 and 32_8. What is the one's digit of their product, written in base 8?

   Answer:

   $5 \times 2 = 12_8$, *so the answer is* $\boxed{2}$.

8. What is the remainder when we divide $78 \times 34 \times 12$ by 11?

Answer:

In base 11 this is $71 \times 31 \times 11$*, so the answer is* $\boxed{1}$.

9. What is the remainder when we divide (99 + 2) $\times$ 17 by 9?

Answer:

The remainder of 99 + 2 is 2 and the remainder of 17 is 8. The remainder of 2×8 is $\boxed{7}$.

10. Suppose we take the product of two numbers whose remainder when divided by 12 is 7. When their product is divided by 12, what is the remainder?

Answer:

$7 \times 7 = 41_{12}$*, so the answer is* $\boxed{1}$.

Exponentiation is just repeated multiplication, so the same thing works for exponentiation. For example, when raising numbers to a power base 10, the one's digit always follows a repeating pattern of length 4 (or some divisor of 4). For example, $7^1 = 7, 7^2 = 49, 7^3 = 63, 7^4 = 341$. So the pattern is $7, 9, 3, 1, 7, 9, 3, 1, \ldots$.

11. What is the one's digit of 11^{10}?

Answer:

1!

12. What is the one's digit of 9^{10}?

Answer:

All even powers have one's digit 1 and all odd powers have one's digit 9. So: $\boxed{1}$.

13. What is the one's digit of 99^{10}?

Answer:

Same as for 9! $\boxed{1}$.

14. What is the one's digit of 2^{2016}?

Answer:

Same as for 2^4*, or* $\boxed{6}$.

15. What is the remainder when 13^{99} is divided by 8?

Answer:

Note that $13 = 15_8$*. Only the 1's digit matters, so it will behave like* 5*. But* $5^2 = 25 = 31_8$*. So the one's digits in base 8 are 5, 1, 5, 1, 5, 1, etc.. The odd powers, such as 99, have a one's digit of* $\boxed{5}$.

16. What is the remainder when 2^{601} is divided by 7?

Answer:

The remainders of the powers of 2 are 2, 4, 1, 2, 4, 1, ..., so the one's digit repeats every 3. Therefore it's the same as for 2^1*, or* $\boxed{2}$.