Principled Programming

Introduction to Coding in Any Imperative Language

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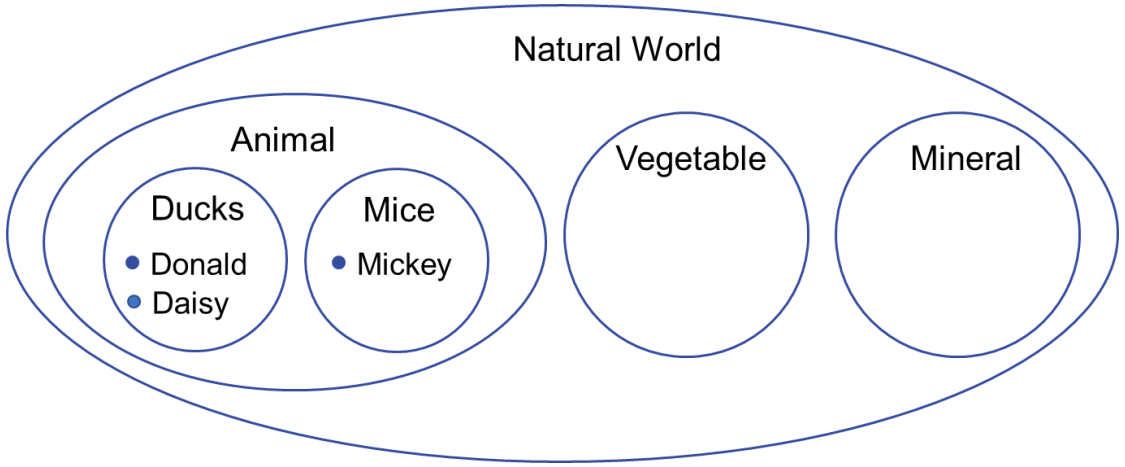
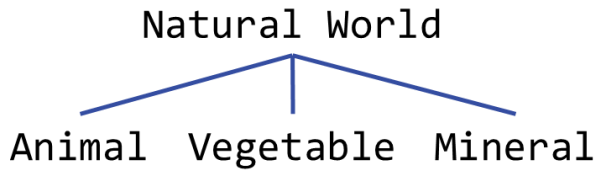
Classes and Objects

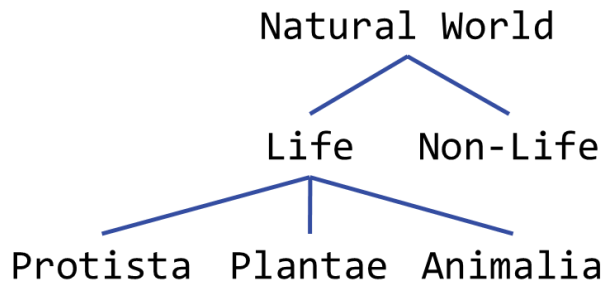
A *taxonomy* is a system of classification. Taxonomies are an essential mechanism for organizing subject matter.

Hierarchical taxonomies in which concepts are organized into tree structures are ubiquitous. In a hierarchy, the most general concept is placed at the root of the tree, and subordinate concepts branch out from there.

Each category is a set of individuals. A Venn diagram depicts categories as nested regions, and individuals as dots.







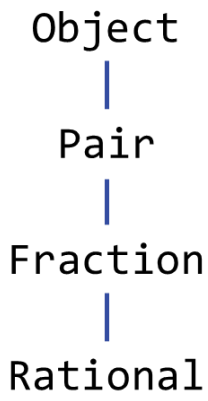
Taxonomic categories in programming are called *classes*, and the individuals of a class are its *objects*. The root category is `Object`.

We illustrate classes and objects by implementing `Pair`, `Fraction`, and `Rational`. Every rational is a fraction, and every fraction is a pair of integers, and every pair is an `Object`.

We then implement `ArrayList<E>`, a parameterized class for representing and manipulating collections of type `E elements`. We use the class `ArrayList<Rational>` to complete code for enumerating rationals.

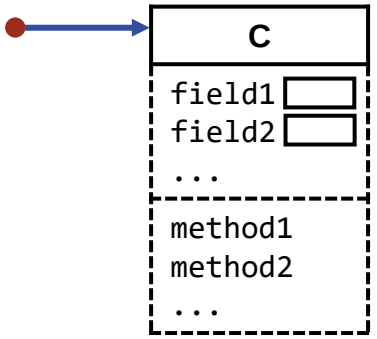
`Because ArrayList<E>` is similar to the library class `HashSet<E>`, it is easy to replace one with the other, and compare their speed. We do so, and demonstrate the dramatic speedup of hash tables over lists.

Finally, in a bit of a double cross, we observe that collections weren'tactually needed for enumerating rationals, and obtain a still-faster implementation without them.



A *class* is a collection of variable declarations and method definitions.

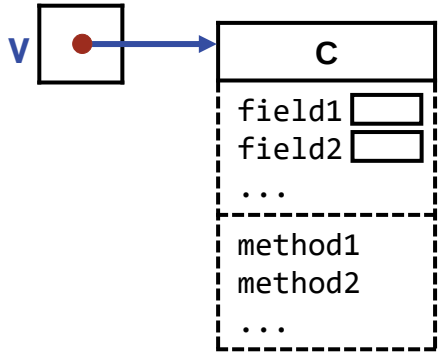
An *object* is a dynamic instantiation of the variables (and methods) of a class whose declarations (and definitions) are not prefixed by the modifier **`static`**.



Such variables are known as *object fields* or *instance variables* (and such methods are known as *instance methods*). Objects and references to them are depicted as shown.

Classes are *types*. If C is a class, a variable v of type C is obtained by executing the declaration:

```
C v;
```



Such a variable can hold a reference to an object of type C.

An object *o* of type C is created by executing the expression

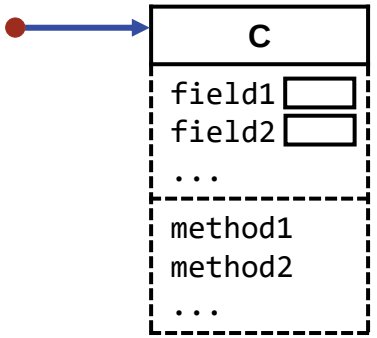
```
new C(…)
```

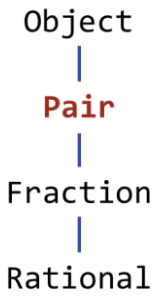
If object *o* has a field `f`, the field is accessed as *o*`.f`.

If object *o* has a method `m`, the method is invoked by *o*`.m(…)`.

If a class is a shape of cookie (with its fields and methods), and objects are the cookies themselves, then `new C(…)` is a cookie-cutter that stamps out new cookies (with instances of C's instance fields and methods.

In contrast, a **`static`** variable (or a **`static`** method) is unique, and is not instantiated for each object. All objects of a class share access to such variables (and methods).





Class definition:

```
class Pair {
   protected int key;
   protected int value;
   /* Constructor. */
      public Pair(int k, int v) { key = k; value = v; }
   /* Access. */
      public int getKey()   { return key; }
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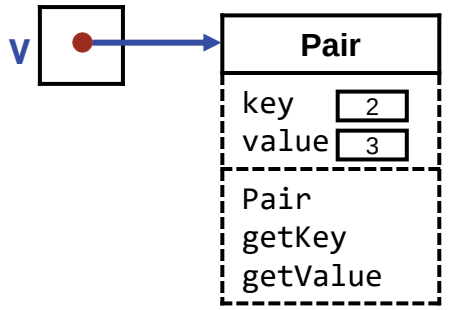
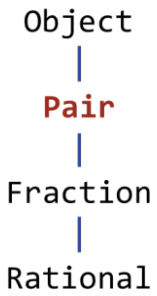
   } /* Pair */
```

Variable declaration (with initialization):

```
Pair v = new Pair(2,3);
```





Execution of the variable declaration (with initialization) in four steps:

1. Create the variable v.

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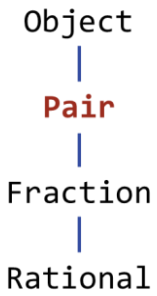

Execution of the variable declaration (with initialization) in four steps:

1. Create the variable v.
2. Create an object of type `Pair`, with default values for its fields.

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class Pair {
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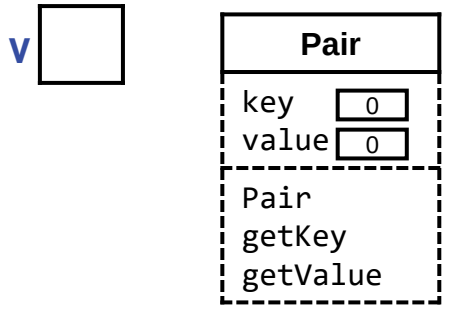

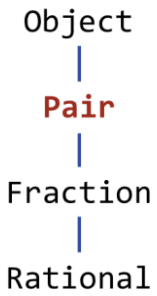

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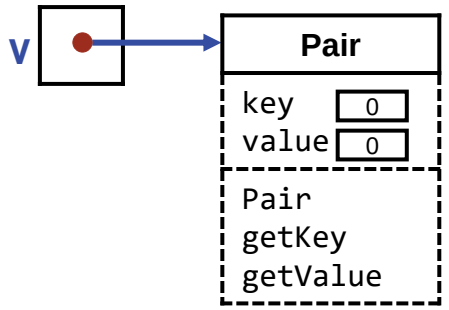

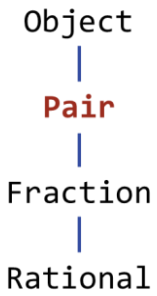

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1. Create the variable v.
2. Create an object of type `Pair`, with default values for its fields.
3. Assign a reference to that object in v.
4. Invoke the constructor `Pair` on the object, which can re-initialize fields.

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   protected int key;
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      public Pair(int k, int v) { key = k; value = v; }
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      public int getKey()   { return key; }
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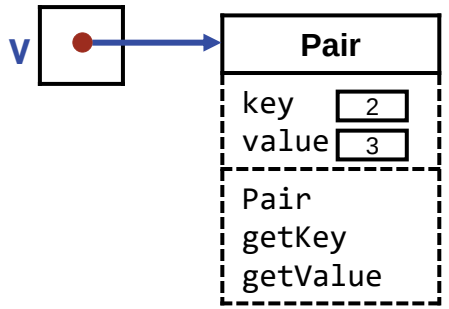
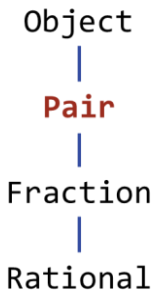
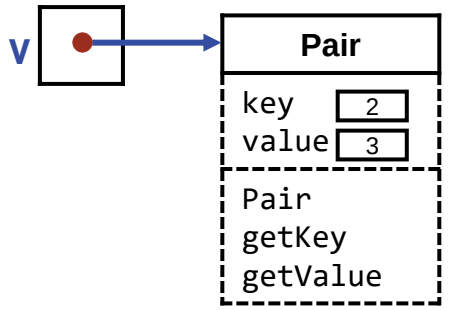


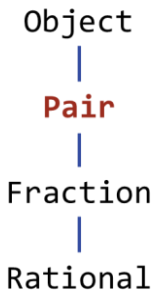


Visibility: Each field and method of a class has visibility **`public`**, **`private`**, or **`protected`**.

```
class Pair {
   protected int key;
   protected int value;
   /* Constructor. */
      public Pair(int k, int v) { key = k; value = v; }
   /* Access. */
      public int getKey()   { return key; }
      public int getValue() { return value; }
   } /* Pair */
```

- **public** fields and methods are globally visible (the default).
- **private** fields and methods are only visible within the class.
- **protected** fields and methods are only visible within the class, or within a subclass of the class, e.g., `Fraction`.





Modifiability: A **private** or **protected** field with a **public** getter is *read-only* outside its scope.

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   protected int key;
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      public Pair(int k, int v) { key = k; value = v; }
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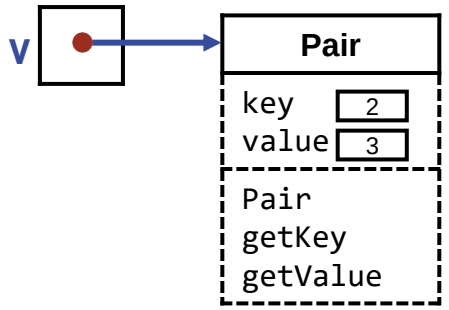
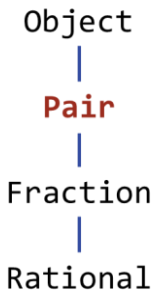
      public int getKey()   { return key; }
      public int getValue() { return value; }
   } /* Pair */
```

- E.g., clients of `Pair` can obtain the components of a `Pair` using the getter, but cannot change those fields. Such an object is said to be *immutable*.





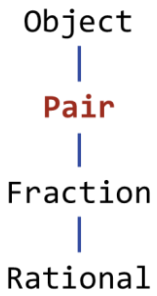
Default `String` representation:

- Every `Pair` is an `Object`, and every `Object` has a default `toString` method.
- However, the String representation provided by that method is not particularly helpful.

Output the `String` representation of an object:

```
System.out.println( v );
```

```
Pair@20293791
```



Overriding definition of `toString` for pairs:

```
class Pair {
   ...
   /* String representation of this. */
      public String toString() { return "<" + key + "," + value + ">"; }
   } /* Pair */
```

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System.out.println( v );
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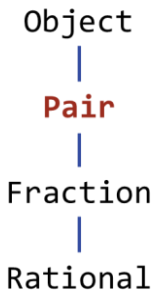
```
<2,3>
```



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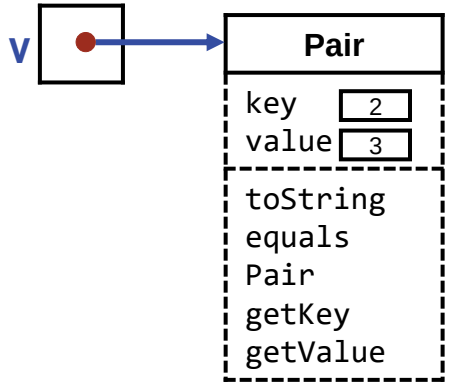

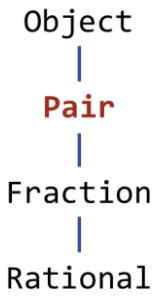

Execution of output statement in three steps:

1. Obtain the value of variable v.

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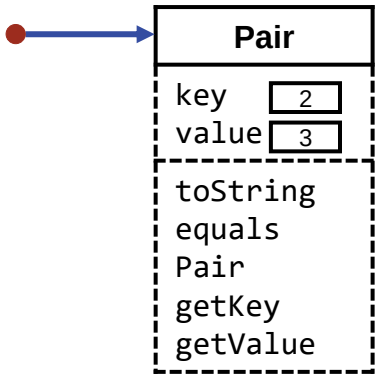

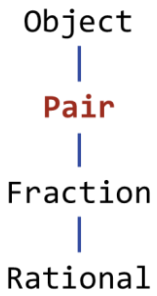

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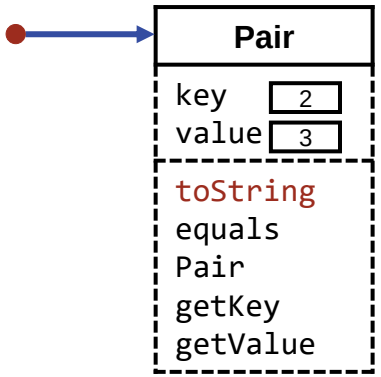

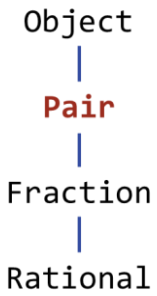

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<2,3>
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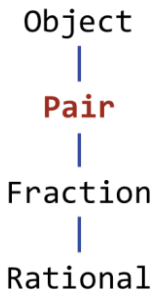


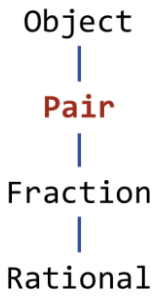
The definition of operator == for objects is identity.

- "Identity" means "exactly the same object".

Demonstrate the difference between identity and equality.

```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1==z2);
System.out.println(z2==z3);
```

```
false
true
```



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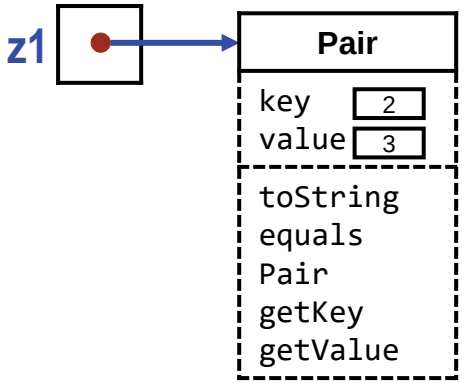

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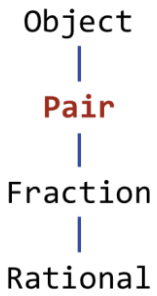

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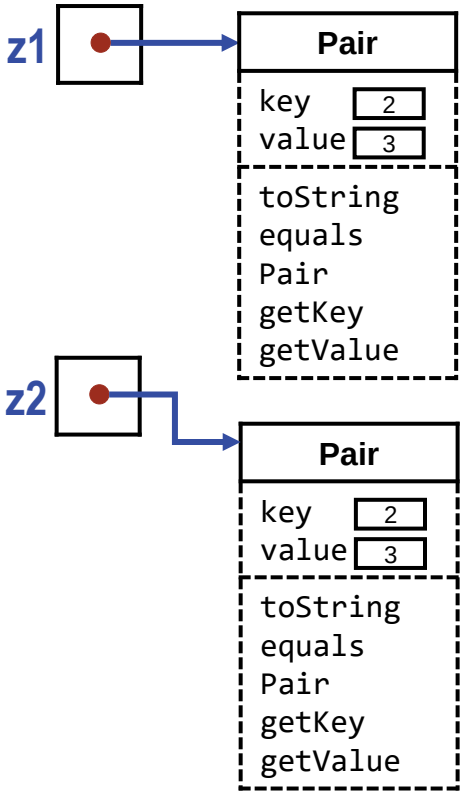

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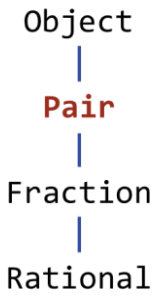

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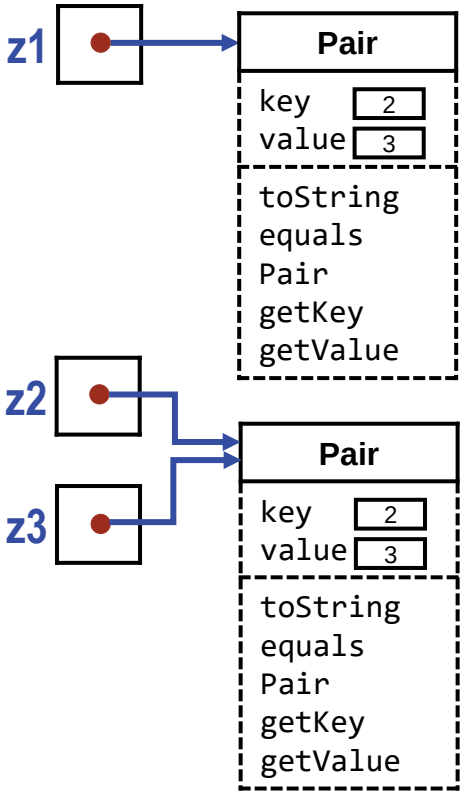

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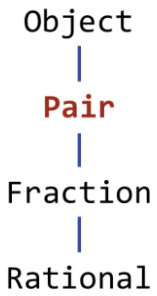

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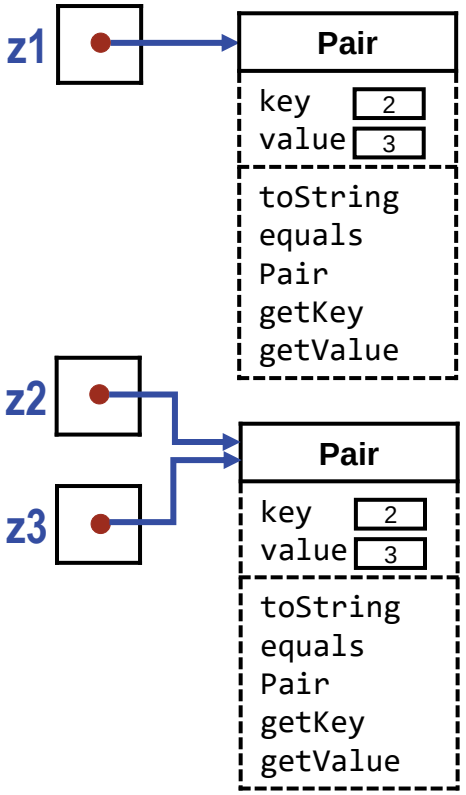

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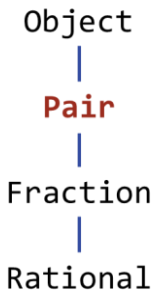
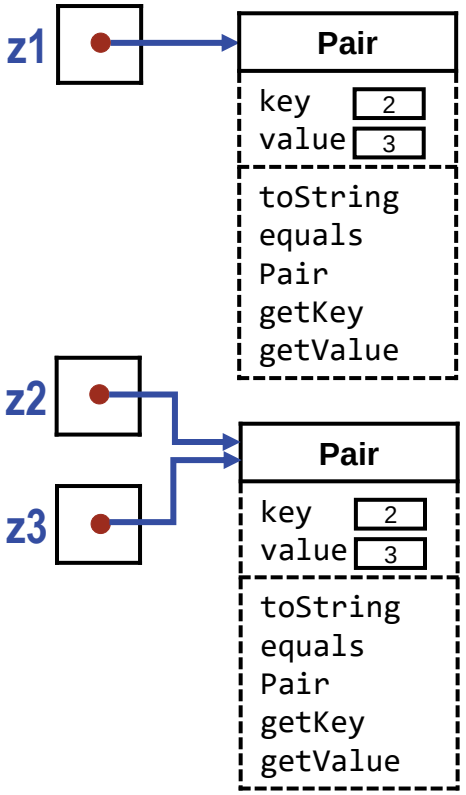
```

```
false
```



The definition of operator == for objects is identity.

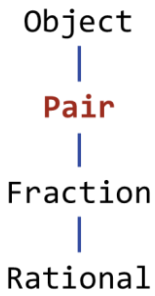
- "Identity" means "exactly the same object".



Demonstrate the difference between identity and equality.

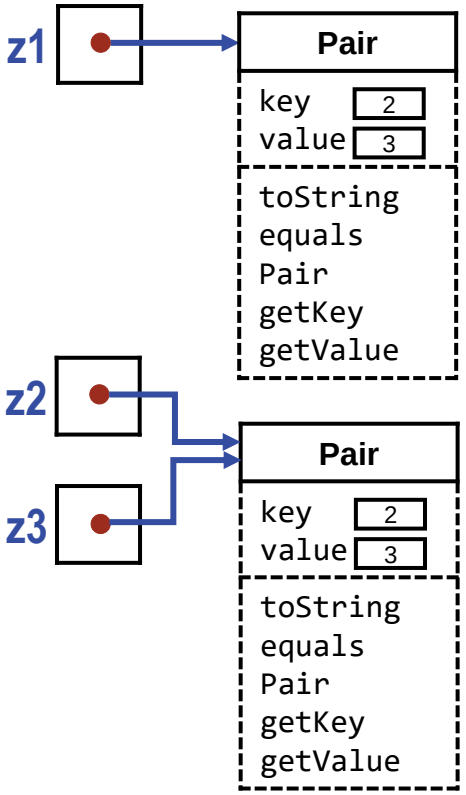
```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1==z2);
System.out.println(z2==z3);
```

```
false
true
```



The default definition of `equals` for `Object` values is also identity.

- "Identity" means "exactly the same object".
- Every `Object` has an `equals` method that can be applied to another `Object` to test "equality", which is user-definable.
- The default definition of method equals in `Object` is identity, i.e., the same as ==.

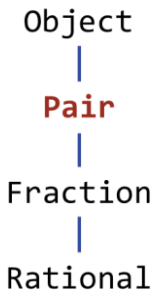


Demonstrate the difference between identity and equality.

```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1.equals(z2));
System.out.println(z2.equals(z3));
```

```
false
true
```

with the default definition of `equals`, i.e., identity



The default definition of `equals` can be overridden.

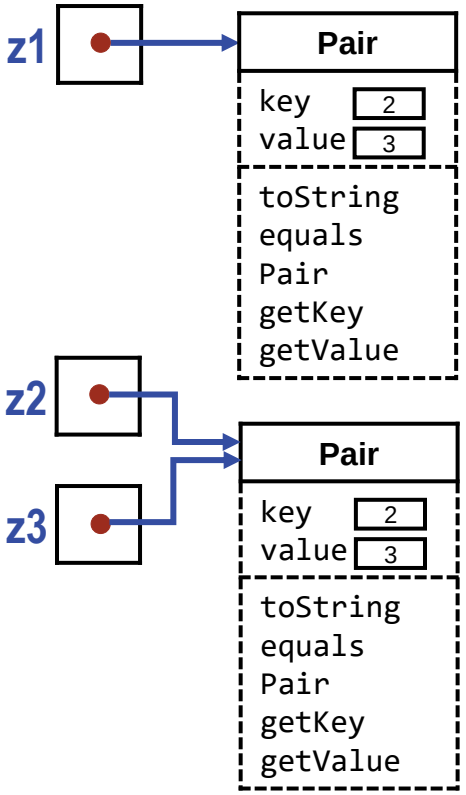
- "Identity" means "exactly the same object".
- Every `Object` has an `equals` method that can be applied to another `Object` to test "equality", which is user-definable.
- The default definition of method equals in `Object` is identity, i.e., the same as ==.
- Unlike the == operator, `equals` can be overridden, e.g., to treat non-identical pairs with equal components as equal.

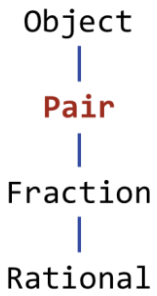
Demonstrate the difference between identity and equality.

```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1.equals(z2));
System.out.println(z2.equals(z3));
```

```
true
true
```

with the overriding definition
of `equals` shown on the next slide





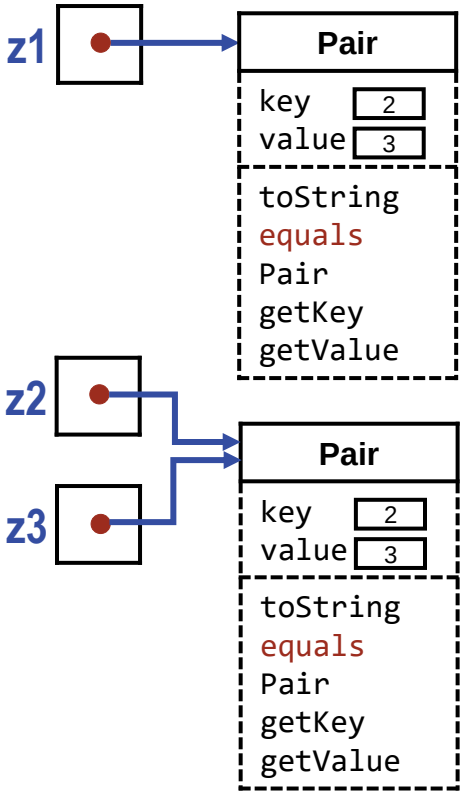
Demonstrate the difference between identity and equality.

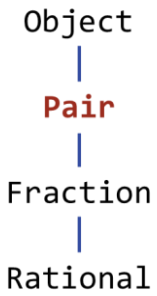
```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1.equals(z2));      true
System.out.println(z2.equals(z3));      true
```

Overriding definition of equals for pairs.

```
class Pair {
   ...
   /* Equality. */
      @Override
      public boolean equals(Object q) {
         if (q==null) return false;
         if (q==this) return true;
         if ( !(q instanceof Pair) ) return false;
         Pair qPair = (Pair)q;
         return (key   == qPair.key) &&
                (value == qPair.value);
         } /* equals */
   } /* Pair */
```

Asks the compiler to warn if the next method definition is not overriding.





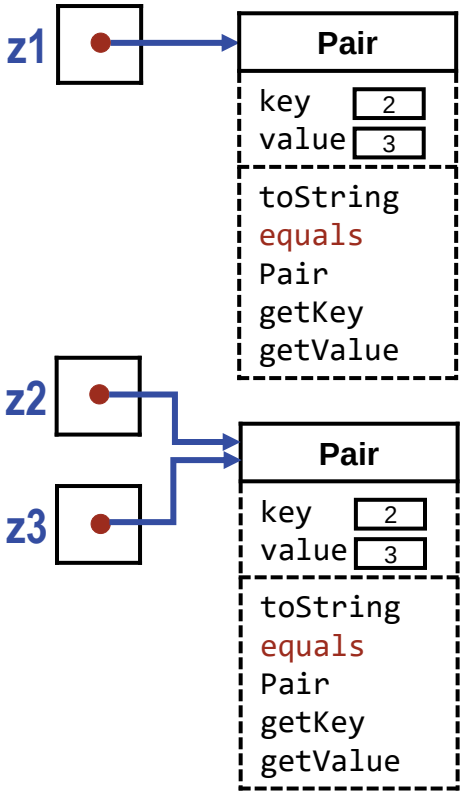
Demonstrate the difference between identity and equality.

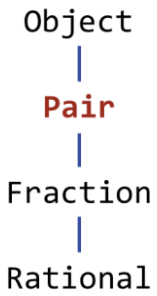
```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1.equals(z2));      true
System.out.println(z2.equals(z3));      true
```

Overriding definition of equals for pairs.

```
class Pair {
   ...
   /* Equality. */
      @Override
      public boolean equals(Object q) {
         if (q==null) return false;
         if (q==this) return true;
         if ( !(q instanceof Pair) ) return false;
         Pair qPair = (Pair)q;
         return (key   == qPair.key) &&
                (value == qPair.value);
         } /* equals */
   } /* Pair */
```

An Object is never equal to no Object.





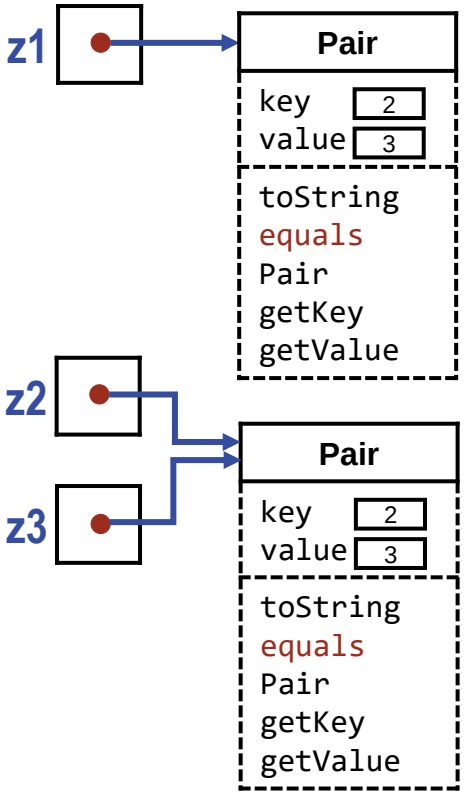
Demonstrate the difference between identity and equality.

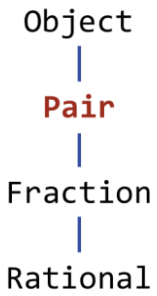
```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1.equals(z2));      true
System.out.println(z2.equals(z3));      true
```

Overriding definition of equals for pairs.

```
class Pair {
   ...
   /* Equality. */
      @Override
      public boolean equals(Object q) {
         if (q==null) return false;
         if (q==this) return true;
         if ( !(q instanceof Pair) ) return false;
         Pair qPair = (Pair)q;
         return (key   == qPair.key) &&
                (value == qPair.value);
         } /* equals */
   } /* Pair */
```

An Object is always equal to itself, e.g. z2 and z3.





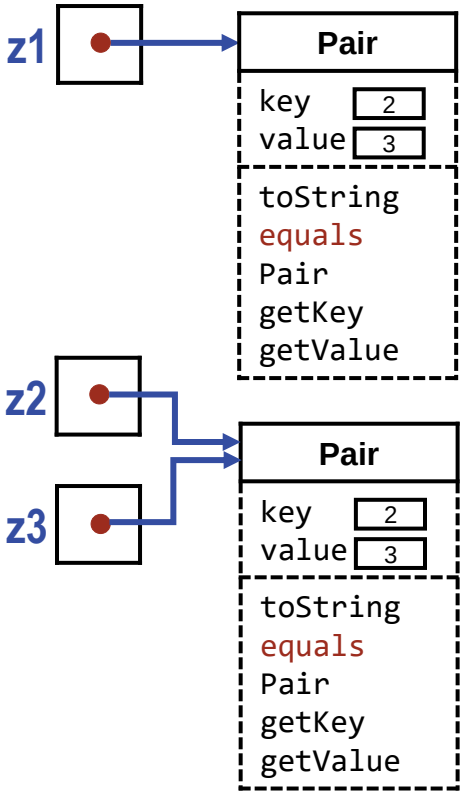
Demonstrate the difference between identity and equality.

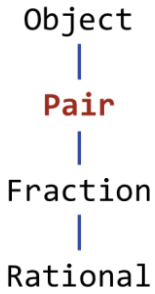
```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1.equals(z2));      true
System.out.println(z2.equals(z3));      true
```

Overriding definition of equals for pairs.

```
class Pair {
   ...
   /* Equality. */
      @Override
      public boolean equals(Object q) {
         if (q==null) return false;
         if (q==this) return true;
         if ( !(q instanceof Pair) ) return false;
         Pair qPair = (Pair)q;
         return (key   == qPair.key) &&
                (value == qPair.value);
         } /* equals */
   } /* Pair */
```

A Pair can only equal another Pair.



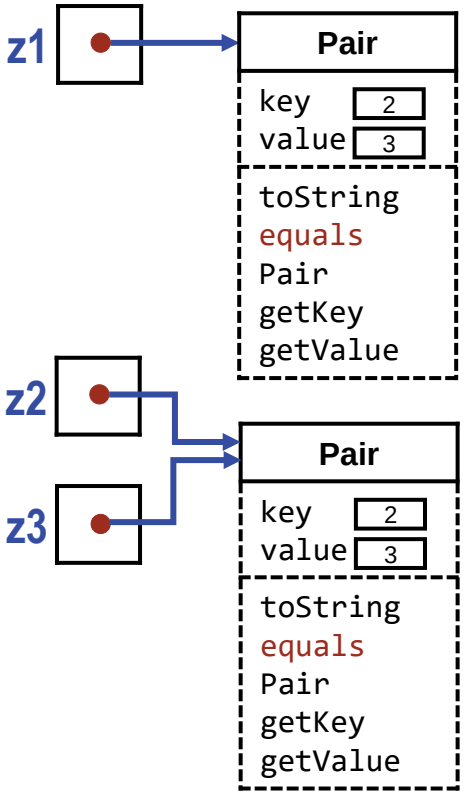


Demonstrate the difference between identity and equality.

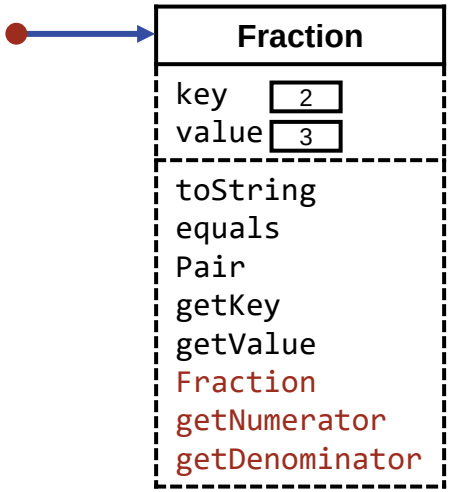
```
Pair z1 = new Pair(2,3);
Pair z2 = new Pair(2,3);
Pair z3 = z2;
System.out.println(z1.equals(z2));      true
System.out.println(z2.equals(z3));      true
```

Overriding definition of `equals` for pairs.

```
class Pair {
   ...
   /* Equality. */
      @Override
      public boolean equals(Object q) {
         if (q==null) return false;
         if (q==this) return true;
         if ( !(q instanceof Pair) ) return false;
         Pair qPair = (Pair)q;
         return (key   == qPair.key) &&
                (value == qPair.value);
         } /* equals */
   } /* Pair */
```



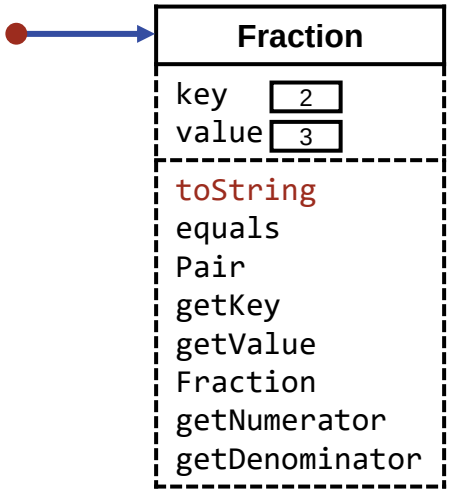
A `Pair` can only equal another `Pair`, and then only when their components are equal, e.g. z1 and z2.



Subclass definition: Fraction

```
class Fraction extends Pair {
   /* Constructor. */
      public Fraction(int numerator, int denominator) {
         super(numerator, denominator); // Apply the Pair constructor.
         assert denominator!=0: "0 denominator";
         }
   /* Access. */
      public int getNumerator()   { return key; }
      public int getDenominator() { return value; }
   } /* Fraction */
```

Getters have direct access to the fields key and value because they are declared **protected** in Pair, a superclass of Fraction.

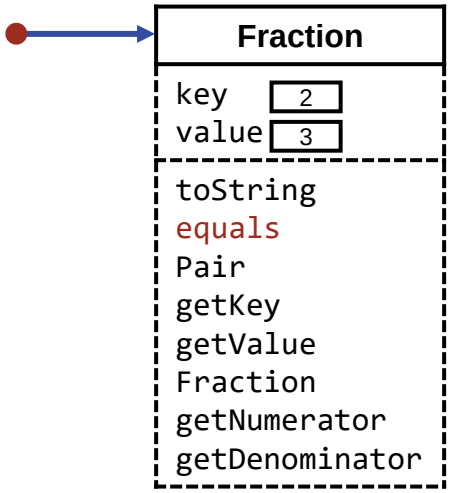


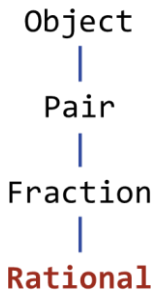
Overriding definition of `toString` for fractions:

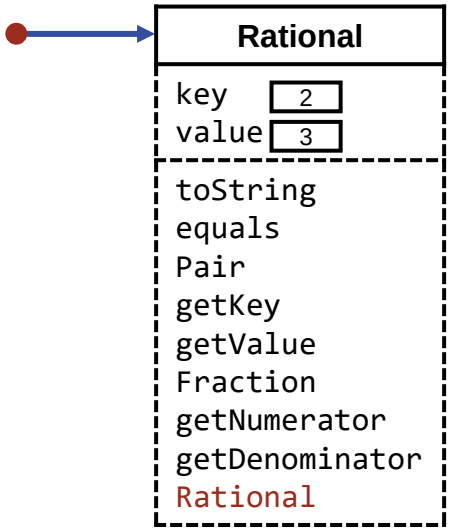
```
class Fraction extends Pair {
   /* Constructor. */
      public Fraction(int numerator, int denominator) {
         super(numerator, denominator); // Apply the Pair constructor.
         assert denominator!=0: "0 denominator";
         }
   /* Access. */
      public int getNumerator()   { return key; }
      public int getDenominator() { return value; }
   /* String representation of this. */
      public String toString() { return key + "/" + value; }
   } /* Fraction */
```

Overriding definition of `equals` for fractions:

- Not needed because two fractions are equal iff they have equal numerators and equal denominators.





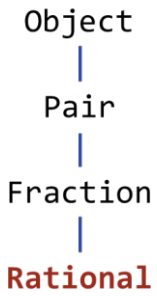


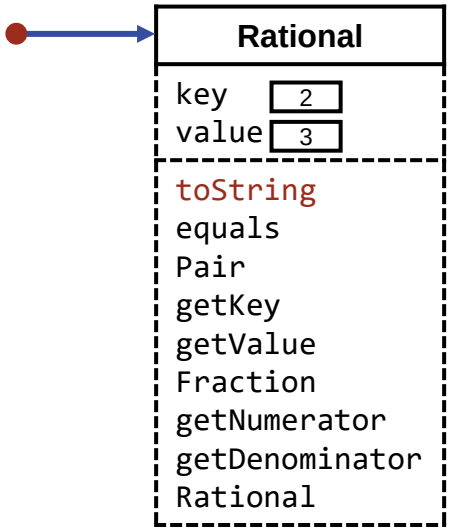
Subclass definition: `Rational`

```
class Rational extends Fraction {
   /* Constructor */
      public Rational(int numerator, int denominator) {
         super(numerator, denominator);    // Apply the Fraction constructor.
         int g = gcd(numerator, denominator);
         key = numerator/g;
         value = denominator/g;
         }
   ...
   } /* Rational */
```

Method gcd not shown.

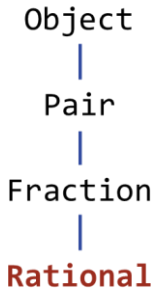
Confirm that the denominator is not zero (checked by Function), and then update the representation to reduced form.





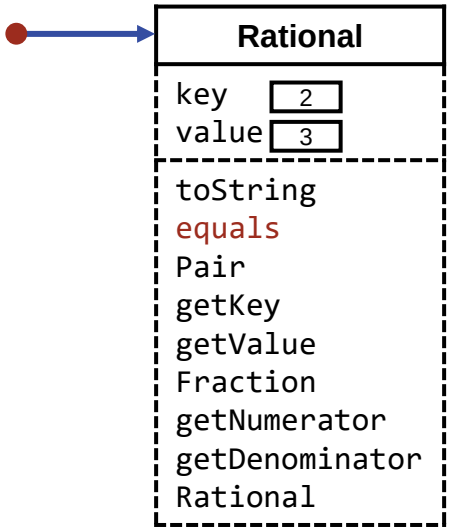
Overriding definition of `toString` for rationals:

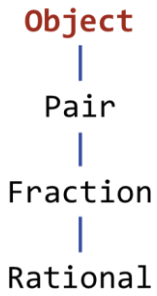
```
class Rational extends Fraction {
   /* Constructor */
      public Rational(int numerator, int denominator) {
         super(numerator, denominator);    // Apply the Fraction constructor.
         int g = gcd(numerator, denominator);
         key = numerator/g;
         value = denominator/g;
         }
   ...
   /* String representation of this. */
      public String toString() {
         if ( value==1 ) return key + "";    // this as int
         else return super.toString();       // this as Fraction
         } /* toString */
   } /* Rational */
```



Overriding definition of `equals` for rationals not needed:

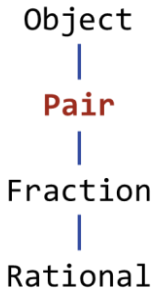
- Not needed because two rationals are equal iff they are `equal` as reduced fractions.

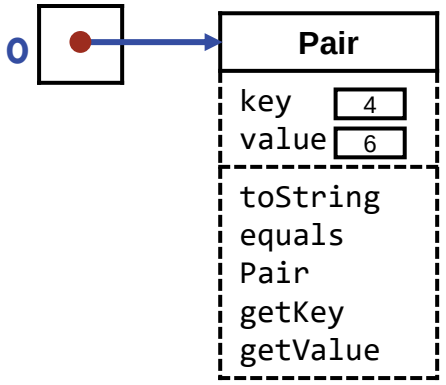




Subtype polymorphism: A variable of class C can be assigned a reference to any object of class C', where C' is either C itself, of C' is a subclass of C, i.e., lower in the class hierarchy.

```
Object o;
```

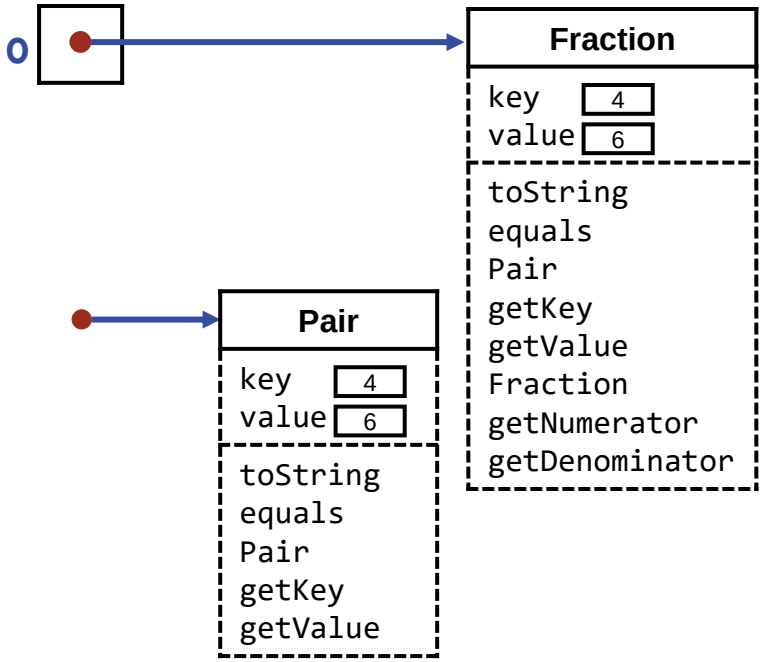




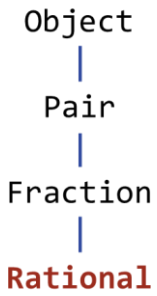
Subtype polymorphism:

```
Object o;
o = new Pair(4,6);
```

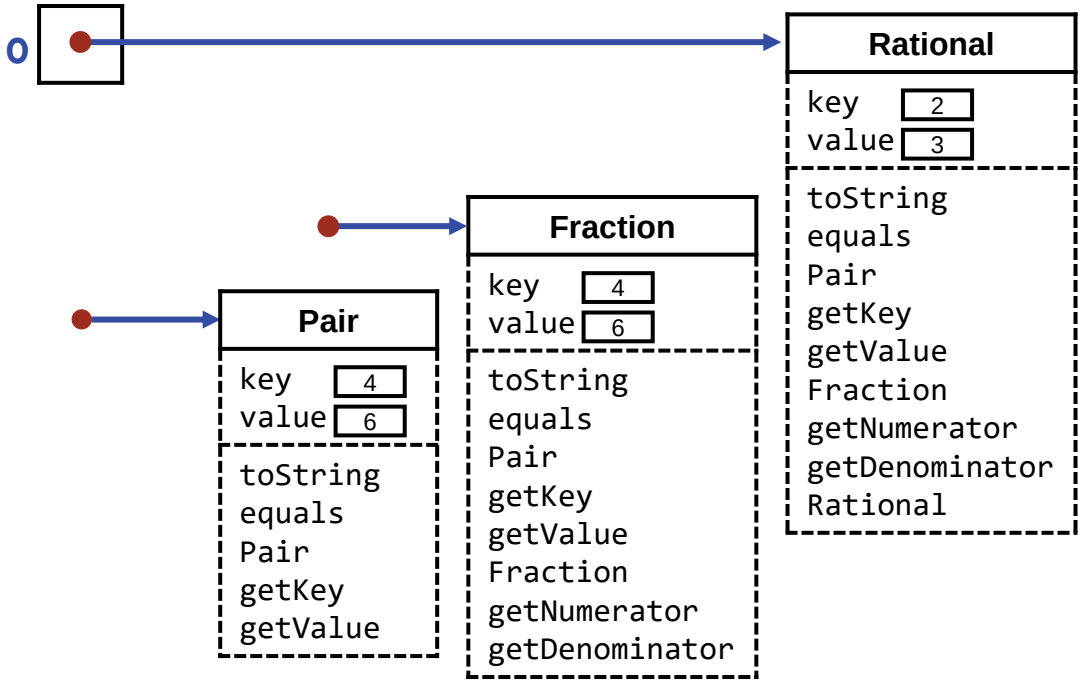
Subtype polymorphism:



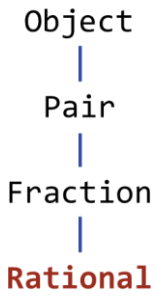

```
Object o;
o = new Pair(4,6);
o = new Fraction(4,6);
```



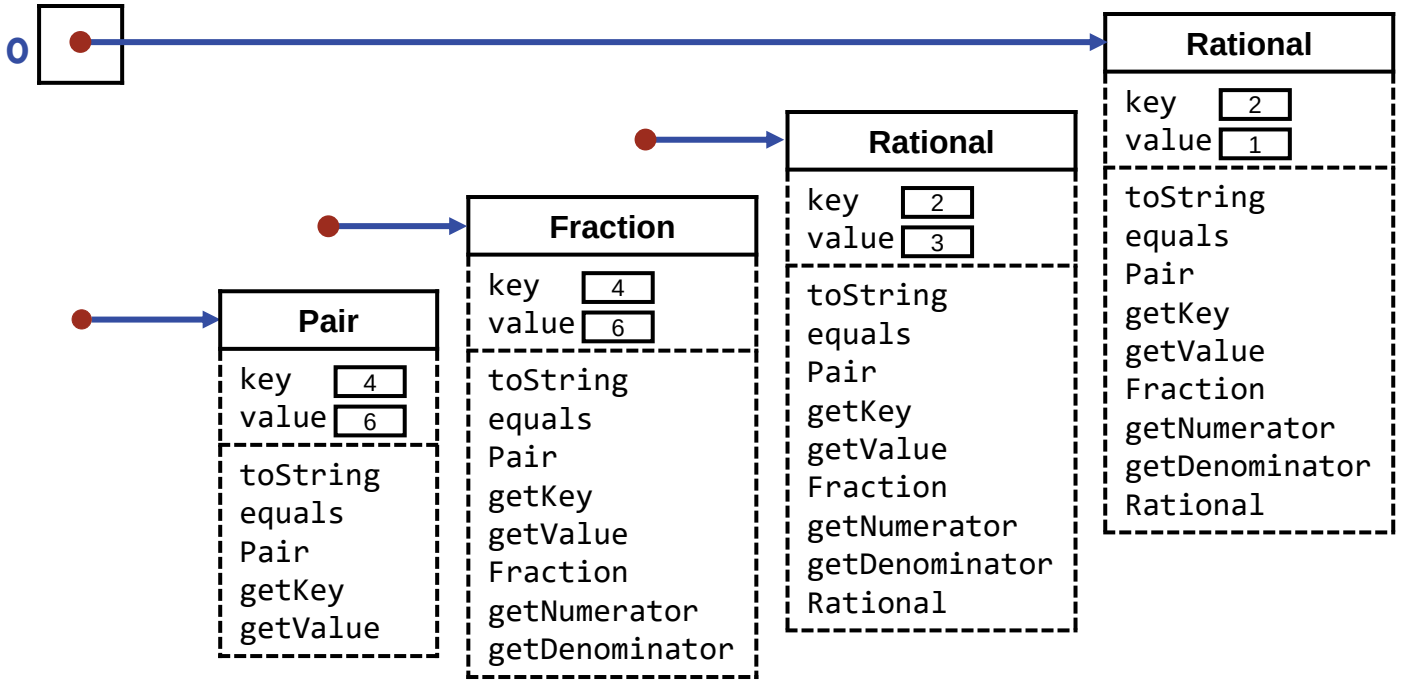
Subtype polymorphism:



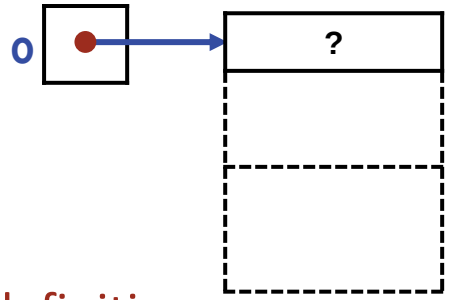

```
Object o;
o = new Pair(4,6);
o = new Fraction(4,6);
o = new Rational(4,6);
```



Subtype polymorphism:

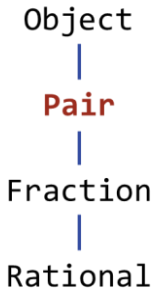


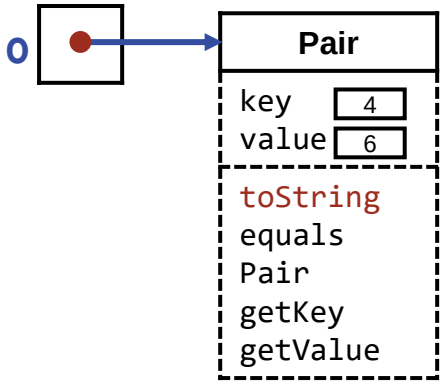

```
Object o;
o = new Pair(4,6);
o = new Fraction(4,6);
o = new Rational(4,6);
o = new Rational(6,3);
```



Dynamic method dispatch: The definition used for any given method invocation depends of the type of the value, not the type of the variable that contains that value.

```
Object o;
```



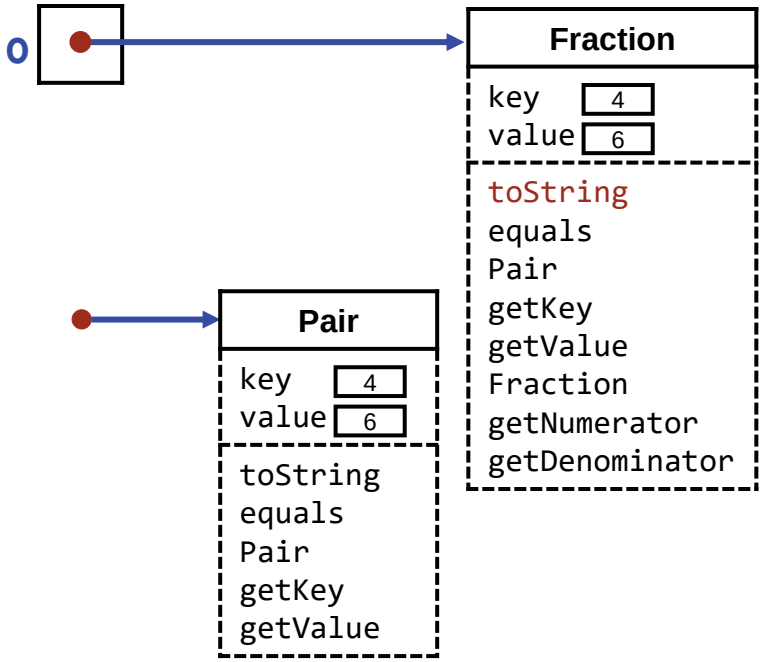


Dynamic method dispatch:

```
Object o;
o = new Pair(4,6);      System.out.println( o );
```

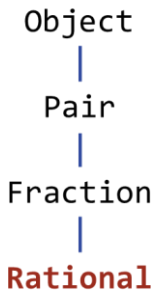
<4,6>

Dynamic method dispatch:

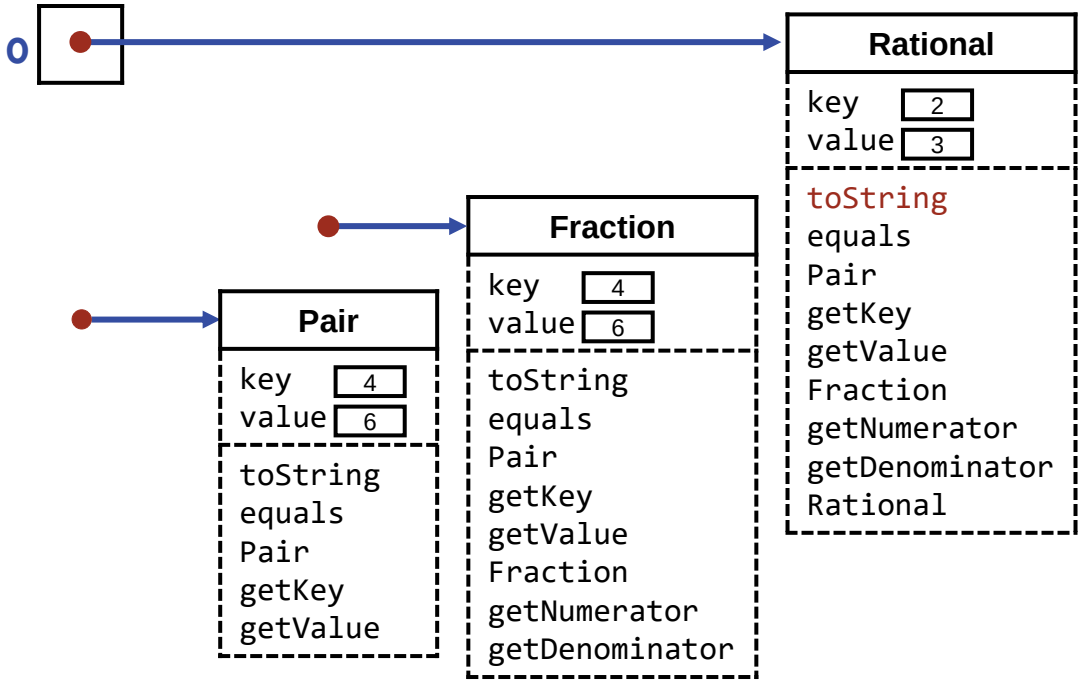



```
Object o;
o = new Pair(4,6);      System.out.println( o );
o = new Fraction(4,6); System.out.println( o );
```

```
<4,6>
4/6
```

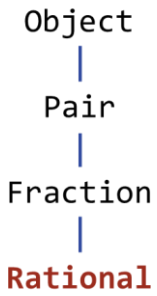


Dynamic method dispatch:

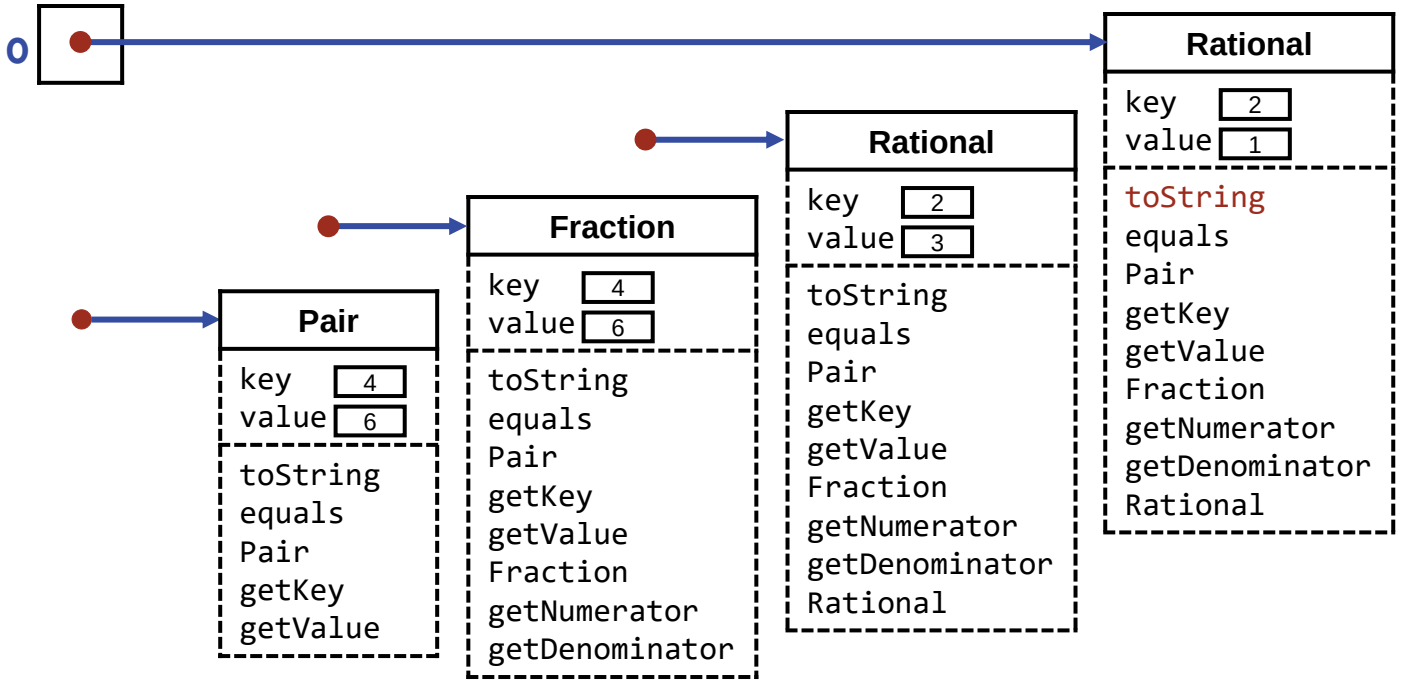



```
Object o;
o = new Pair(4,6);      System.out.println( o );
o = new Fraction(4,6); System.out.println( o );
o = new Rational(4,6); System.out.println( o );
```

```
<4,6>
4/6
2/3
```

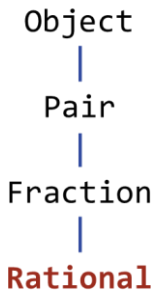


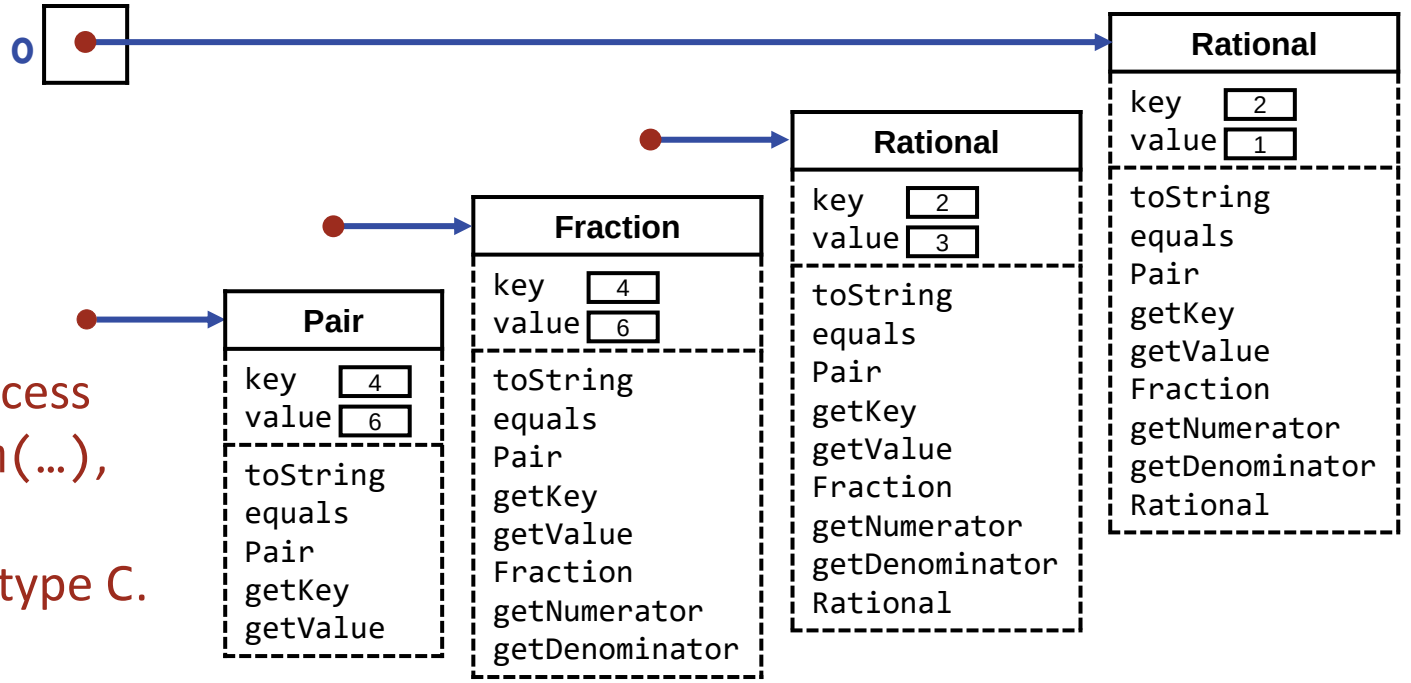
Dynamic method dispatch:




```
Object o;
o = new Pair(4,6);      System.out.println( o );
o = new Fraction(4,6);  System.out.println( o );
o = new Rational(4,6);  System.out.println( o );
o = new Rational(6,3);  System.out.println( o );
```

```
<4,6>
4/6
2/3
2
```

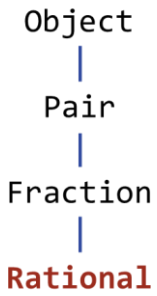


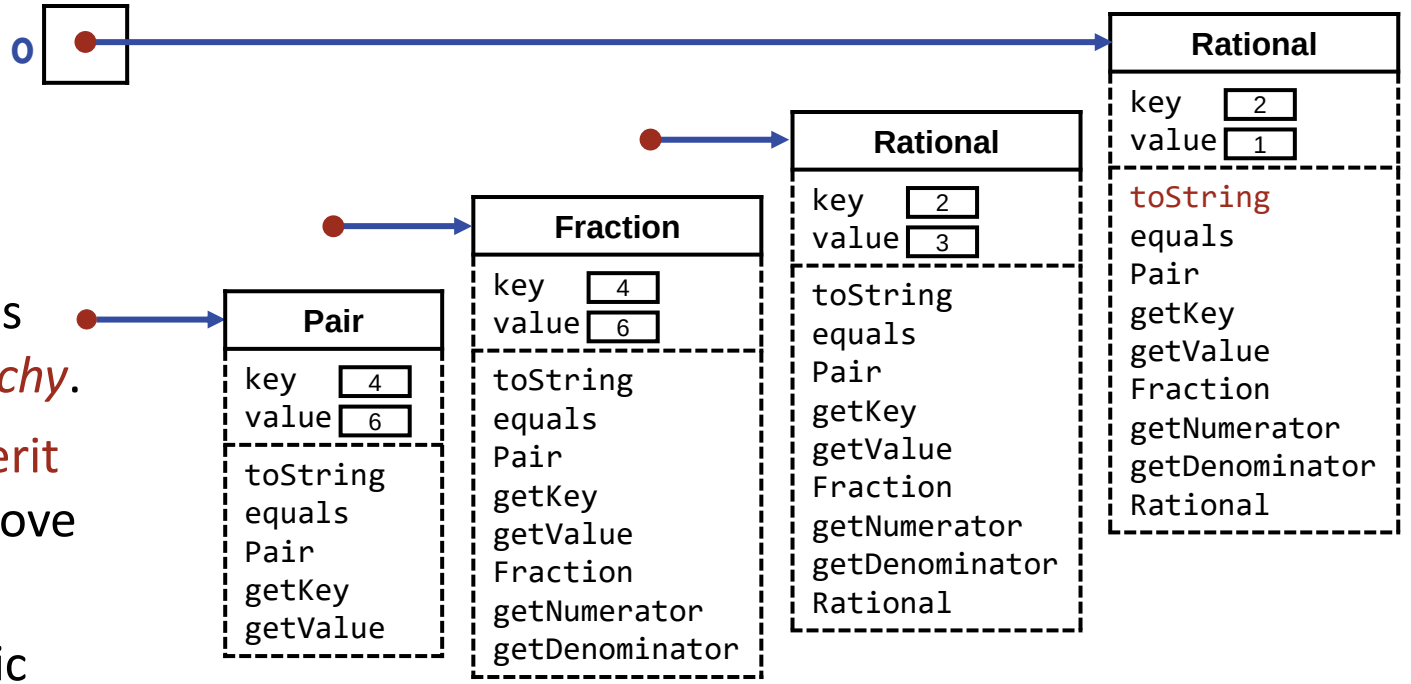


Subtype polymorphism caveat:

If variable v has type C, a field access `v.f`, or a method invocation `v.m(…)`, requires that field `f` or method `m` necessarily exist in any object of type C.

```
Object   o = new Pair(4,6);     System.out.println(o.getKey());       // Illegal.
Pair     p = new Pair(4,6);     System.out.println(p.getKey());       // Legal.
         p = new Pair(2,3);     System.out.println(p.getNumerator()); // Illegal.
Fraction r = new Fraction(4,6); System.out.println(r.getNumerator()); // Legal.
Rational q = new Rational(4,6); System.out.println(q.getNumerator()); // Legal.
```





Inheritance: The class hierarchy is also called the *inheritance hierarchy*.

Objects of class C are said to inherit all fields *f* of superclasses of C above it in the hierarchy.

They also inherit the most specific (overriding) version of method *m* defined either in class C, or in one of C's superclasses, i.e., the first definition of *m* found in a traversal from C up to `Object` in the hierarchy.

Class definition:

```
class ArrayList {
   private int A[];   // ArrayList elements are in A[0..size-1].
   private int size;  // The default value is 0.

   ...
```

Data representation is private, i.e., hidden to clients.

The type of an `ArrayList` element is **`int`** (for now).

Class definition:

```
class ArrayList {
   private int A[];   // ArrayList elements are in A[0..size-1].
   private int size;  // The default value is 0.

   ...
```

Two overloaded constructors: One for a specific initial capacity, the other for a default capacity.

Class definition:

```
class ArrayList {
   private int A[];   // ArrayList elements are in A[0..size-1].
   private int size;  // The default value is 0.

   /* Constructors. */
      public ArrayList( int m ) {
         if ( m<0 ) throw new IllegalArgumentException();
         A = new int[m];
         }

      public ArrayList() { this( 20 /* DEFAULT_SIZE */ );

   ...
```

A getter for the read-only field `size`, and a predicate to test for an empty list.

Class definition:

```
class ArrayList {
   private int A[];   // ArrayList elements are in A[0..size-1].
   private int size;  // The default value is 0.

   /* Constructors. */
      public ArrayList( int m ) {
         if ( m<0 ) throw new IllegalArgumentException();
         A = new int[m];
         }

      public ArrayList() { this( 20 /* DEFAULT_SIZE */ );

   /* Size. /
      public int size() { return size; }
      public boolean isEmpty() { return size==0; }

   ...
```

Get and set the k-th element of the list. Set returns the old k-th value of the list.

```
/* Access. */
   public int get(int k) {
      checkBoundExclusive(k);
      return A[k];
      }
   public int set(int k, int v) {
      checkBoundExclusive(k);
      int old = A[k];
      A[k] = v;
      return old;
      }

...
```

Detect an index that is too large. Negative indices are caught by normal subscript bounds check.

```
/* Access. */
   public int get(int k) {
      checkBoundExclusive(k);
      return A[k];
      }
   public int set(int k, int v) {
      checkBoundExclusive(k);
      int old = A[k];
      A[k] = v;
      return old;
      }

/* Utility */
   private void checkBoundExclusive( int k ) {
      if (k>=size) throw new IndexOutOfBoundsException( "≥size" );
      }
   private void checkBoundInclusive( int k ) {
      if (k>size) throw new IndexOutOfBoundsException( ">size" );
      }

...
```

Insert an elements v in list, either at end or at position k. Increase capacity if necessary.

```
/* Insertion / Deletion. */
   public void add(int v) {
      if ( size==A.length ) ensureCapacity( size+1 );
      A[size] = v; size++;
      }
   public void add(int k, int v) {
      checkBoundInclusive(k);
      if ( size==A.length ) ensureCapacity( size+1 );
      for (int j=size; j>k; j--) A[j] = A[j-1];
      A[k] = v;
      size++;
      }

...
```

Remove an element from list at end or at position k.

```
/* Insertion / Deletion. */
   public void add(int v) {
      if ( size==A.length ) ensureCapacity( size+1 );
      A[size] = v; size++;
      }
   public void add(int k, int v) {
      checkBoundInclusive(k);
      if ( size==A.length ) ensureCapacity( size+1 );
      for (int j=size; j>k; j--) A[j] = A[j-1];
      A[k] = v;
      size++;
      }
   public int remove(int k) {
      checkBoundExclusive(k);
      int old = A[k];
      size--;
      for (int j=k; j<size; j++) A[j] = A[j+1];
      return old;
      }

...
```

Increase the list's capacity by updating A to refer to a copy of A with double the length.

```
/* Capacity. */
   public void ensureCapacity( int minCapacity) {
      int currentLength = A.length;
      if ( minCapacity > currentLength ) {
         int B[] = new int[Math.max(2*currentLength, minCapacity)];
         for (int k=0; k<size; k++) B[k] = A[k];
         A = B;
         }
      }

...
```

Find the location of a value v in the list. Test for membership of a value v in the list.

```
/* Capacity. */
   public void ensureCapacity( int minCapacity) {
      int currentLength = A.length;
      if ( minCapacity > currentLength ) {
         int B[] = new int[Math.max(2*currentLength, minCapacity)];
         for (int k=0; k<size; k++) B[k] = A[k];
         A = B;
         }
      }

/* Membership. */
   public int indexOf(int v) {
      int k = 0; while ( (k<n) && (v!=A[k]) ) k++;
      if ( k==n ) return -1; else return k;
      }
   public boolean contains(int v) {
      return indexOf(v)!=-1;
      }

} /* ArrayList */
```

Enumeration of rationals: Recall this incomplete code from Enumeration Patterns.

```
/* Output reduced positive fractions, i.e., positive rationals. */
   /* set reduced = { }; */
   int d = 0;
   while ( true ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         /* Let z be the reduced form of the fraction (r+1)/(c+1). */
            int g = gcd(r+1, c+1);
            /* rational z = ⟨(r+1)/g, (c+1)/g⟩; */
         if ( /* z is not an element of reduced */ ) {
            System.out.println( /* z */ );
            /* reduced = reduced ∪ {z}; */
            }
         r--;
         }
      d++;
      }
```

Enumeration of rationals: We can adopt `Rational` as the type of rational z.

```
/* Output reduced positive fractions, i.e., positive rationals. */
   /* set reduced = { }; */
   int d = 0;
   while ( true ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         /* Let z be the reduced form of the fraction (r+1)/(c+1). */
            Rational z = new Rational(r+1, c+1);

         if ( /* z is not an element of reduced */ ) {
            System.out.println( z );
            /* reduced = reduced ∪ {z}; */
            }
         r--;
         }
      d++;
      }
```

Similarly, we would like to adopt `ArrayList` as the type of the set `reduced`, but cannot do so because as currently written it is a collection of **`int`** items, not `Rational` items.

Enumeration of rationals: We can adopt `Rational` as the type of rational z.

```
/* Output reduced positive fractions, i.e., positive rationals. */
   /* set reduced = { }; */
   int d = 0;
   while ( true ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         /* Let z be the reduced form of the fraction (r+1)/(c+1). */
            Rational z = new Rational(r+1, c+1);

         if ( /* z is not an element of reduced */ ) {
            System.out.println( z );
            /* reduced = reduced ∪ {z}; */
            }
         r--;
         }
      d++;
      }
```

Enumeration of rationals: We need an `ArrayList` of `Rational` items.

This could be done by:

- Cloning the `ArrayList` of **`int`** implementation, and adapting the clone to be a collection of `Rational` elements (ugh!), or
- Parameterizing `ArrayList` to be `ArrayList<E>`, a collection of elements of arbitrary object type `E`, and then instantiating it as `ArrayList<Rational>`, a collection of `Rational` elements (far better!).

A parametrized class definition is called a *generic class*.

Generic class definition:

The type of an ArrayList element is parameterized as **E**.

```
class ArrayList<E> {
   private E A[];     // ArrayList elements are in A[0..size-1].
   private int size;  // The default value is 0.

   /* Constructors. */
      public ArrayList( int m ) {
         if ( m<0 ) throw new IllegalArgumentException();
         A = (E[]) new Object[m];          }

      public ArrayList() { this( 20 /* DEFAULT_SIZE */ );

   /* Size. /
      public int size() { return size; }
      public boolean isEmpty() { return size==0; }

   ...
```

An array of arbitrary objects is created, and is cast to the type of A.

```
/* Access. */
   public E get(int k) {
      checkBoundExclusive(k);
      return A[k];
      }
   public E set(int k, E v) {
      checkBoundExclusive(k);
      E old = A[k];
      A[k] = v;
      return old;
      }

/* Utility */
   private void checkBoundExclusive( int k ) {
      if (k>=size) throw new IndexOutOfBoundsException( "≥size" );
      }
   private void checkBoundInclusive( int k ) {
      if (k>size) throw new IndexOutOfBoundsException( ">size" );
      }

...
```

```
/* Insertion / Deletion. */
   public void add(E v) {
      if ( size==A.length ) ensureCapacity( size+1 );
      A[size] = v; size++;
      }
   public void add(int k, E v) {
      checkBoundInclusive(k);
      if ( size==A.length ) ensureCapacity( size+1 );
      for (int j=size; j>k; j--) A[j] = A[j-1];
      A[k] = v;
      size++;
      }
   public E remove(int k) {
      checkBoundExclusive(k);
      E old = A[k];
      size--;
      for (int j=k; j<size; j++) A[j] = A[j+1];
      A[size] = null; // Garbage-collection assist.
      return old;
      }
...
```

To be explained in Garbage Collection discussion.

An array of arbitrary objects is created, and is cast to the type of B.

```
/* Capacity. */
   public void ensureCapacity( int minCapacity) {
      int currentLength = A.length;
      if ( minCapacity > currentLength ) {
         E B[] = (E[]) new Object[Math.max(2*currentLength, minCapacity)];
         for (int k=0; k<size; k++) B[k] = A[k];
         A = B;
         }
      }

/* Membership. */
   public int indexOf(E v) {
      int k = 0; while ( (k<n) && !v.equals(A[k]) ) k++;
      if ( k==n ) return -1; else return k;
      }
   public boolean contains(E v) {
      return indexOf(v)!=-1;
      }

} /* ArrayList */
```

Use the `equals` method of the element type rather than ==.

Enumeration of rationals: Returning to the incomplete code for enumerating rationals.

```
/* Output reduced positive fractions, i.e., positive rationals. */
   /* set reduced = { }; */
   int d = 0;
   while ( true ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         /* Let z be the reduced form of the fraction (r+1)/(c+1). */
            Rational z = new Rational(r+1, c+1);
         if ( /* z is not an element of reduced */ ) {
            System.out.println( z );
            /* reduced = reduced ∪ {z}; */
            }
         r--;
         }
      d++;
      }
```

Enumeration of rationals: We declare `reduced` to have type `ArrayList<Rational>`.

```
/* Output reduced positive fractions, i.e., positive rationals. */
   ArrayList<Rational> reduced = new ArrayList();
   int d = 0;
   while ( true ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         /* Let z be the reduced form of the fraction (r+1)/(c+1). */
            Rational z = new Rational(r+1, c+1);
         if ( !reduced.contains(z) ) {
            System.out.println( z );
            reduced.add(z);
            }
         r--;
         }
      d++;
      }
```

Enumeration of rationals: and obtain the correct output.

1
2
1/2
3
1/3 ← 2/2 omitted
4
3/2
2/3
1/4
5
1/5 ← 4/2, 3/3, and 2/4 omitted
6
5/2
4/3
3/4
2/5
1/6
7
5/3 ← 6/2 omitted
3/5 ← 4/4 omitted
1/7 ← 2/6 omitted
etc.

Uniformity:

- In some languages, all values are uniformly objects of a class.
- In other languages, there is a distinction between *primitive values* and objects of a class.
- Primitive values, e.g., values of types **`int`**, **`long`**, **`float`**, **`double`**, **`boolean`**, and **`char`**, fit conveniently into variables of standard sizes.
- In contrast, objects are accessed via references. The object reference has a standard size, but the object itself doesn't.
- In the interest of efficiency, but at the expense of complexity, Java offers two worlds, one in which values are primitive, and the other in which values are objects.
- Crossing back and forth between the two worlds is a bit complicated, and will be addressed next.

Class definition: Recall the definition of class `Pair`.

```
class Pair {
   protected int key;
   protected int value;
   /* Constructor. */
      public Pair(int k, int v) { key = k; value = v; }
   /* Access. */
      public int getKey()   { return key; }
      public int getValue() { return value; }
   } /* Pair */
```

Generic class definition: It too can be made generic so we can have pairs of any object types.

```
class Pair<K,V> {
   protected K key;
   protected V value;
   /* Constructor. */
      public Pair(K k, V v) { key = k; value = v; }
   /* Access. */
      public K getKey() { return key; }
      public V getValue() { return value; }
   ...
   } /* Pair<K, V> */
```

Generic class definition: It too can be made generic so we can have pairs of any object type.

```
class Pair<K,V> {
   ...
   /* Equality. */
      @Override
      public boolean equals(Object q) {
         if (q==null) return false;
         if (q==this) return true;
         if ( !(q instanceof Pair) ) return false;
         Pair qPair = (Pair)q;
         return key.equals(qPair.key) &&
                value.equals(qPair.value);
         } /* equals */
   } /* Pair */
```

Uses the `equals` methods of the component types (which need not be the same) rather than ==.

Pairs of any object type. The generic class `Pair<K,V>` can be instantiated with any object types for **K** and **V**.

For example, each of the following is a valid declaration:

```
Pair<Fraction, Fraction> ff;
Pair<Fraction, Rational> fr;
Pair<Fraction, Object> fo;
Pair< Pair<Fraction, Fraction>, Pair<Rational, Rational> ffrr;
```

but the following is not a valid declaration:

```
Pair<int, int> ii;
```

because **int** is a primitive type, not an object type.

We deal with this next.

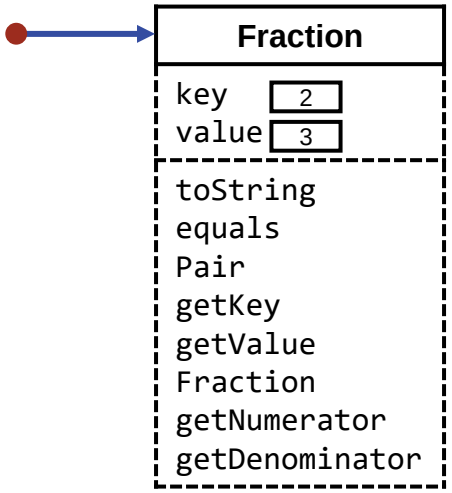
Boxed values.

There is an object type for each primitive type. Values of those types are called *boxed primitive values*:

```
Integer
Long
Float
Double
Boolean
Char
```

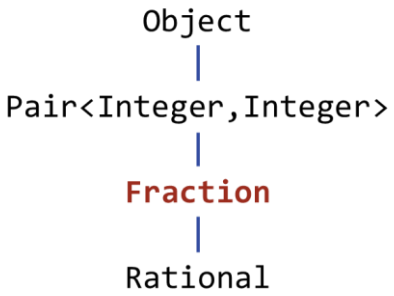
Java attempts to box and unbox values fully automatically, but you need to know that it is going on.

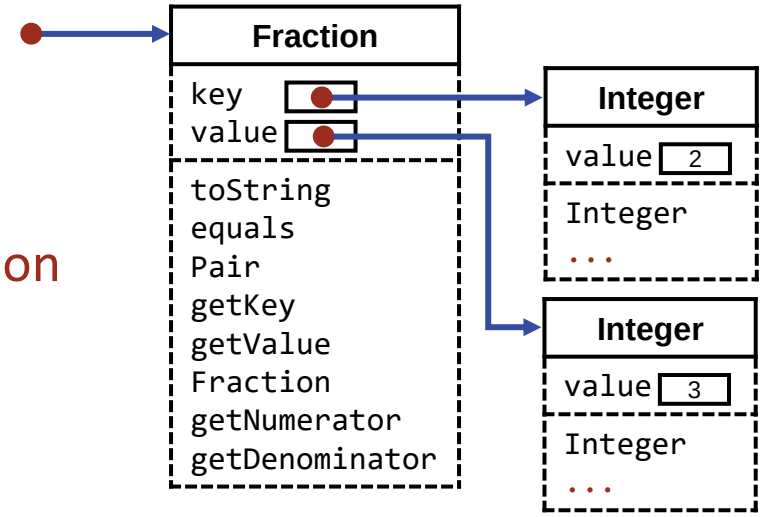
For example, if we were to change `Pair` to be the generic class `Pair<K,V>`, the definition of class `Function` would no longer be correct, as explained next.



Subclass definition: Recall the definition of `Fraction`.

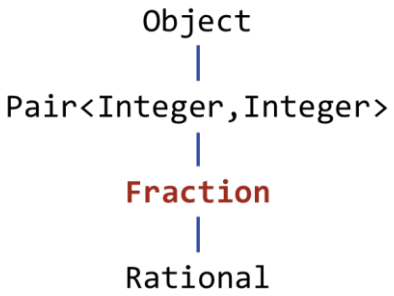
```
class Fraction extends Pair {
   /* Constructor. */
      public Fraction(int numerator, int denominator) {
         super(numerator, denominator); // Apply the Pair constructor.
         assert denominator!=0: "0 denominator";
         }
   /* Access. */
      public int getNumerator()   { return key; }
      public int getDenominator() { return value; }
   } /* Fraction */
```

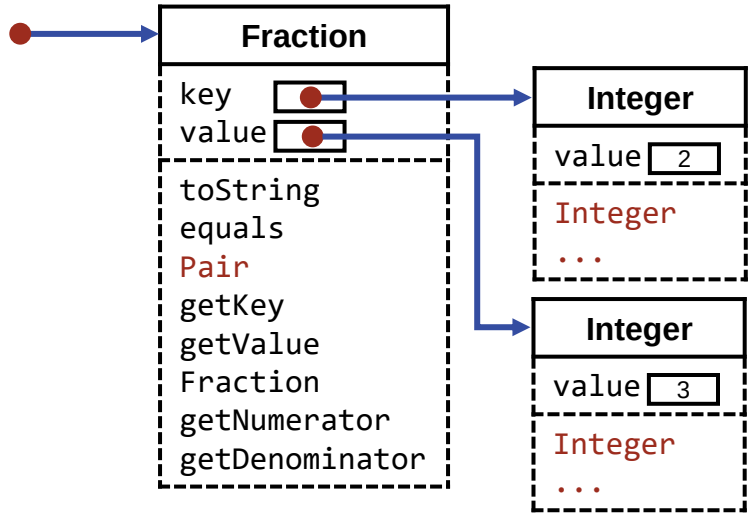




Subclass definition: Since `Pair` is now a generic class, `Fraction` must instantiate it with component types.

```
class Fraction extends Pair<Integer,Integer> {
   /* Constructor. */
      public Fraction(int numerator, int denominator) {
         super(numerator, denominator); // Apply the Pair constructor.
         assert denominator!=0: "0 denominator";
         }
   /* Access. */
      public int getNumerator()   { return key; }
      public int getDenominator() { return value; }
   } /* Fraction */
```

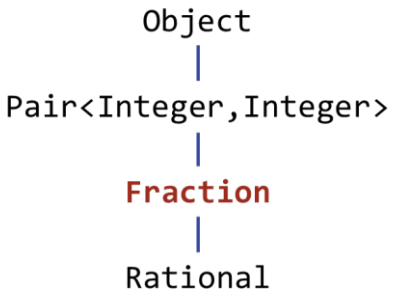


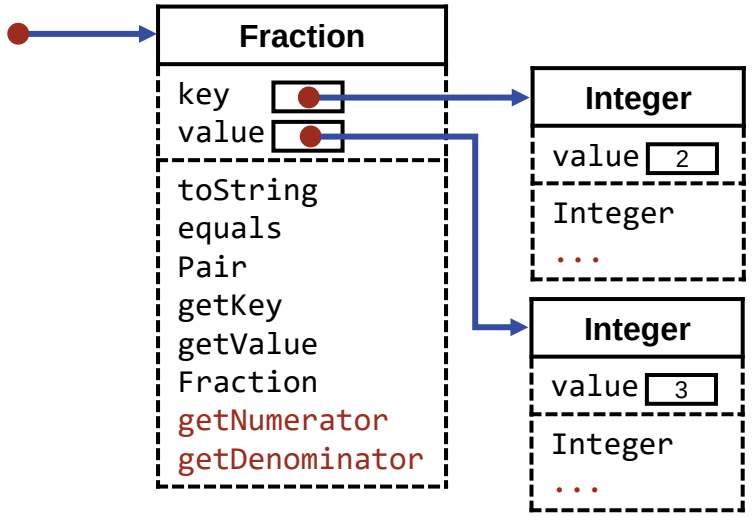


Subclass definition: Auto-boxing and auto-unboxing occurs between **int** values and Integer values.

```
class Fraction extends Pair<Integer,Integer> {
   /* Constructor. */
      public Fraction(int numerator, int denominator) {
         super(numerator, denominator); // Apply the Pair constructor.
         assert denominator!=0: "0 denominator";
         }
   /* Access. */
      public int getNumerator()   { return key; }
      public int getDenominator() { return value; }
   } /* Fraction */
```

Auto-boxing of int parameters numerator and denominator occurs when they are passed to the Pair constructor since it expects Integer arguments..

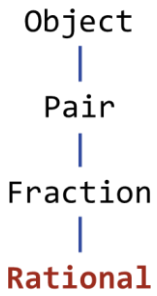


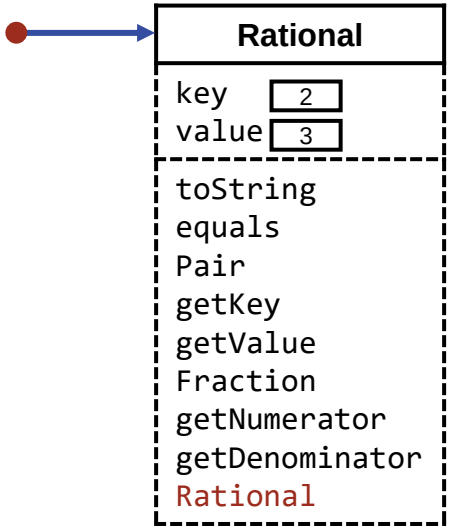


Subclass definition: Auto-boxing and unboxing occurs between **`int`** values and `Integer` values.

```
class Fraction extends Pair<Integer,Integer> {
   /* Constructor. */
      public Fraction(int numerator, int denominator) {
         super(numerator, denominator); // Apply the Pair constructor.
         assert denominator!=0: "0 denominator";
         }
   /* Access. */
      public int getNumerator()   { return key; }
      public int getDenominator() { return value; }
   } /* Fraction */
```

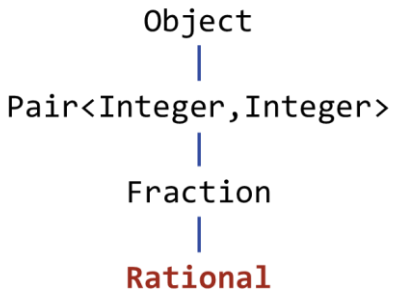
Auto-unboxing of the `Integer` `key` and `value` fields occurs when they returned as the values of the getters .

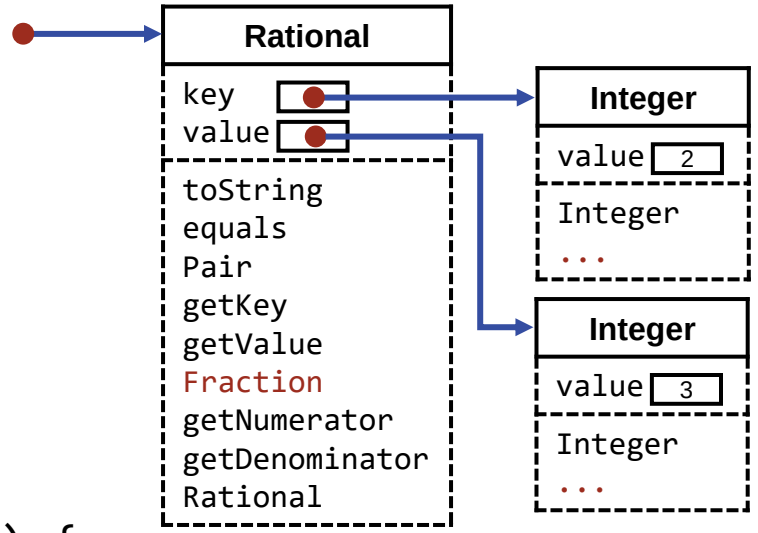




Subclass definition: Similarly, recall the definition of `Rational`.

```
class Rational extends Fraction {
   /* Constructor */
      public Rational(int numerator, int denominator) {
         super(numerator, denominator);    // Apply the Fraction constructor.
         int g = gcd(numerator, denominator);
         key = numerator/g;
         value = denominator/g;
         }
   ...
   } /* Rational */
```

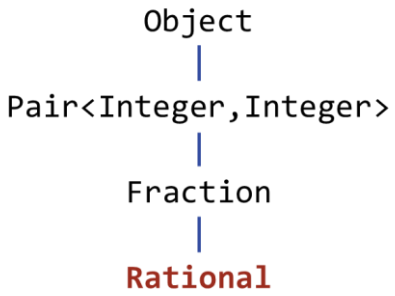


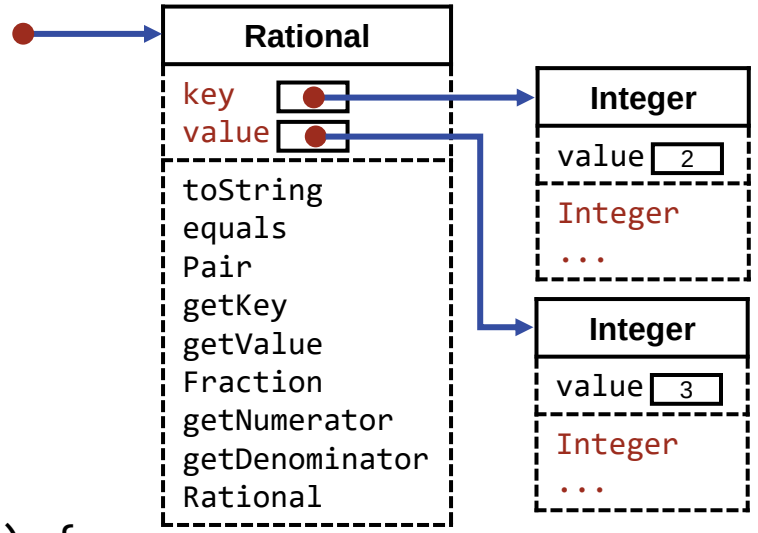


Subclass definition: Since `Fraction` inherits from `Pair<Integer,Integer>`, so too does `Rational`.

```
class Rational extends Fraction {
   /* Constructor */
      public Rational(int numerator, int denominator) {
         super(numerator, denominator);    // Apply the Fraction constructor.
         int g = gcd(numerator, denominator);
         key = numerator/g;
         value = denominator/g;
         }
   ...
   } /* Rational */
```

No auto-boxing of `int` parameters `numerator` and `denominator` occurs when they are passed to the `Function` constructor because it expects two `int` arguments. They are auto-boxed when it invokes the `Pair` constructor.





Subclass definition: Since `Fraction` inherits from `Pair<Integer,Integer>`, so too does `Rational`.

```
class Rational extends Fraction {
   /* Constructor */
      public Rational(int numerator, int denominator) {
         super(numerator, denominator);    // Apply the Fraction constructor.
         int g = gcd(numerator, denominator);
         key = numerator/g;
         value = denominator/g;
         }
   ...
   } /* Rational */
```

Auto-boxing of the computed `int` values `numerator/g` and `denominator/g` occurs when they are assigned to the `Integer` `key` and `value` fields.

Polymorphism: Four kinds have been illustrated.

- **Subtype polymorphism**, where an object of one class is treated as an instance of any of its superclasses. Thus, a variable declared to have a given class as its type may contain a value of that class, or of any of its subclasses. Dynamic dispatch selects the appropriate code for a method invocation based on the specific type of the given value.
- **Parametric polymorphism**, where a class definition is abstracted with respect to one or more class parameters, resulting in a generic class, which can be viewed as a cookie cutter that stamps out classes (i.e., generic-class instances).
- **Conversion**, where an expression of one type occurs in a context that expects a value of a different type, and it is implicitly converted to the required type. Other conversions are explicit, e.g., casts. Another term for conversion is coercion.
- **Overloading**, where different methods have the same name, and the appropriate definition is chosen based on the number and types of arguments in the invocation.

e.g., the object constructed by `Rational(2,3)` can be treated as a `Rational`, `Fraction`, `Pair`, or `Object`.

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e.g., a variable declared to have type Fraction can be assigned a `Fraction` or `Rational`, but it cannot be assigned a `Pair` or `Object`.

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e.g., the code executed for `toString` depends on the type of the object, e.g., `Rational`.

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e.g., `ArrayList<E>` or `Pair<K,V>`.

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e.g., `ArrayList<Rational>` or `Pair<Integer,Integer>`.

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e.g., the boxing of an `int` in the `Fraction` constructor, and the unboxing of an `Integer` in the Rational getters.

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e.g., the cast `(E[])` in the statement A = `(E[]) new Object[m];` in the `ArrayList<E>` constructor.

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e.g., `ArrayList<E>` has two constructors, one with no parameter, and the other with one parameter. It also has two add methods, one with one parameter, and the other with two parameters.

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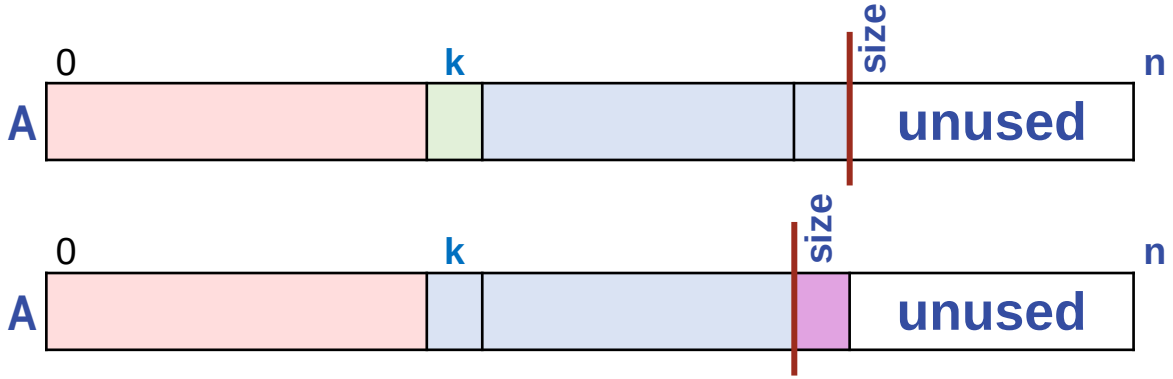
Garbage Collection. An object dies when it can no longer be accessed in the program.

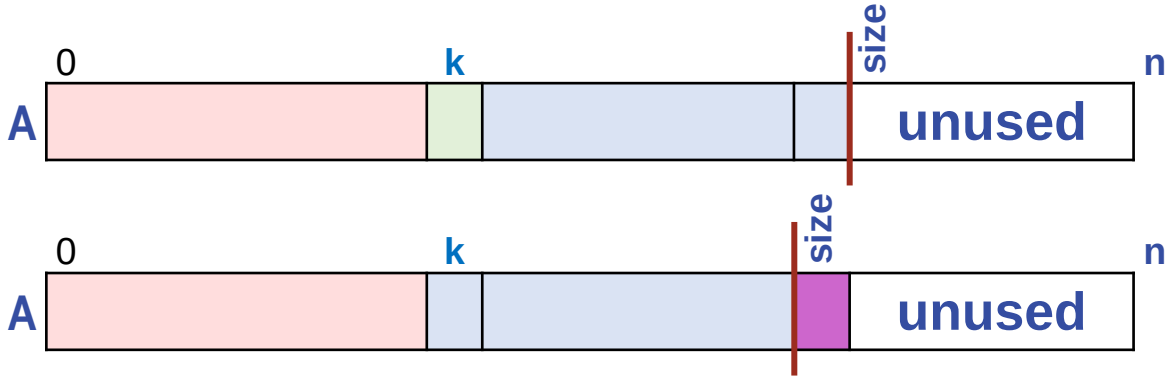
- Objects consume space in computer memory.
- Space consumed by objects that can no longer be accessed can be reclaimed automatically by a mechanism (that runs behind the scene) called *garbage collection*.
- Normally, you don't have to think about such matters. However, you should be aware that retaining a gratuitous reference to an object can cause it to be needlessly retained.
- By itself, one such object is no big concern. But if it is at the beginning of a chain of references from one object to another, then that one gratuitous reference can be the cause of an unbounded number of needlessly-retained other objects, which is of concern.
- This is why we make sure that an `ArrayList<E>` retains no gratuitous references to objects in the unused suffix of the array.
- We explain how this works next. It is a bit subtle, but is instructive.



Garbage Collection. Recall the definition of `remove` in `ArrayList<E>`.

```
public E remove(int k) {
        checkBoundExclusive(k);
        E old = A[k];
        size--;
        for (int j=k; j<size; j++) A[j] = A[j+1];
        A[size] = null; // Garbage-collection assist.
        return old;
        }
```

The left shift of (blue) values overwrites the (green) value in `A[k]` that is to be removed from the collection. It was a reference to some object, and if this was the only reference to that object, it can be garbage collected. The value at `A[k]` being removed is *not* the issue.



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        A[size] = null; // Garbage-collection assist.
        return old;
        }
```

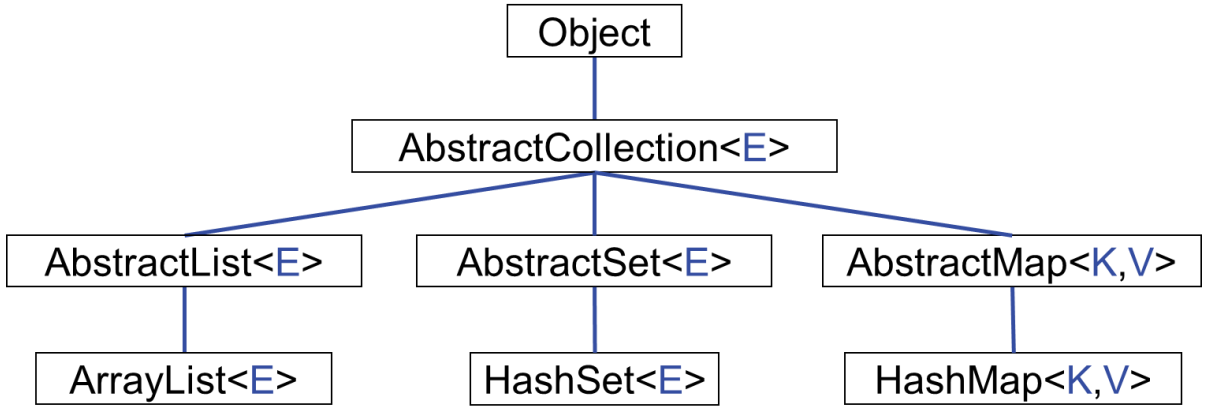
The last (blue) value in the collection, which was originally in `A[size-1]`, shifts left but because the shift is effected by *copying,* the original instance of that value (violet) would also remain as `A[size]`, the first element of the unused array suffix. It is this violet instance of the value that we nullify. Note that the object referred to by the violet reference can not yet be collected because a reference to it remains in `A[size-1]`. However, if and when *that* reference is removed or is overwritten with a new value by `set`, the object in question will be collectable.

Libraries: Classes that you can learn and use.

Libraries are extensions of the core language. The standard library includes:

- `Object`, the root of the class inheritance hierarchy. All other classes are subclasses of `Object`, and inherit methods from it.
- `Math`, a class that contains built-in mathematical functions as **`static`** methods.
- `String`, the class for sequences of Unicode characters. String constants, e.g., `"a String"`, are references to `String` objects that contain the given sequence of characters.
- `Integer`, `Boolean`, etc., classes for the boxed primitive values.

Libraries: The library `java.util` contains many useful classes, including these for collections:



Class `ArrayList<E>`, which we (partially) implemented ourselves, appears in the inheritance hierarchy as a second cousin of `HashSet<E>`, a familial relationship that we would have obtained by writing:

```
import java.util.*;
public class ArrayList<E> extends AbstractList<E> { ... }
```

An abstract class provides names and parameter types of methods that its non-abstract subclasses must implement, but not the method bodies themselves. This allows its subclasses to have completely different implementations, but be interchangeable.

Enumeration of rationals: Recall our code for enumerating rationals using `ArrayList<E>`.

```
/* Output reduced positive fractions, i.e., positive rationals. */
   ArrayList<Rational> reduced = new ArrayList();
   int d = 0;
   while ( true ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         /* Let z be the reduced form of the fraction (r+1)/(c+1). */
            Rational z = new Rational(r+1, c+1);
         if ( !reduced.contains(z) ) {
            System.out.println( z );
            reduced.add(z);
            }
         r--;
         }
      d++;
      }
```

Enumeration of rationals: To use HashSet<E> instead, we only need to change one line.

```
/* Output reduced positive fractions, i.e., positive rationals. */
   HashSet<Rational> reduced = new HashSet();
   int d = 0;
   while ( true ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         /* Let z be the reduced form of the fraction (r+1)/(c+1). */
            Rational z = new Rational(r+1, c+1);
         if ( !reduced.contains(z) ) {
            System.out.println( z );
            reduced.add(z);
            }
         r--;
         }
      d++;
      }
```

The text of the contains and add invocations is unchanged, but the methods that are actually invoked change radically, i.e., from the ArrrayList<E> implementations to the HashSet<E> implementations.

Enumeration of rationals: and provide a hash function for `Pair<K,V>`.

```
class Pair<K,V> {
   ...
   /* HashFunction. */
      @Override
      public int hashCode() {
         return key.hashCode() + value.hashCode();
         } /* hashCode */
   } /* Pair */
```

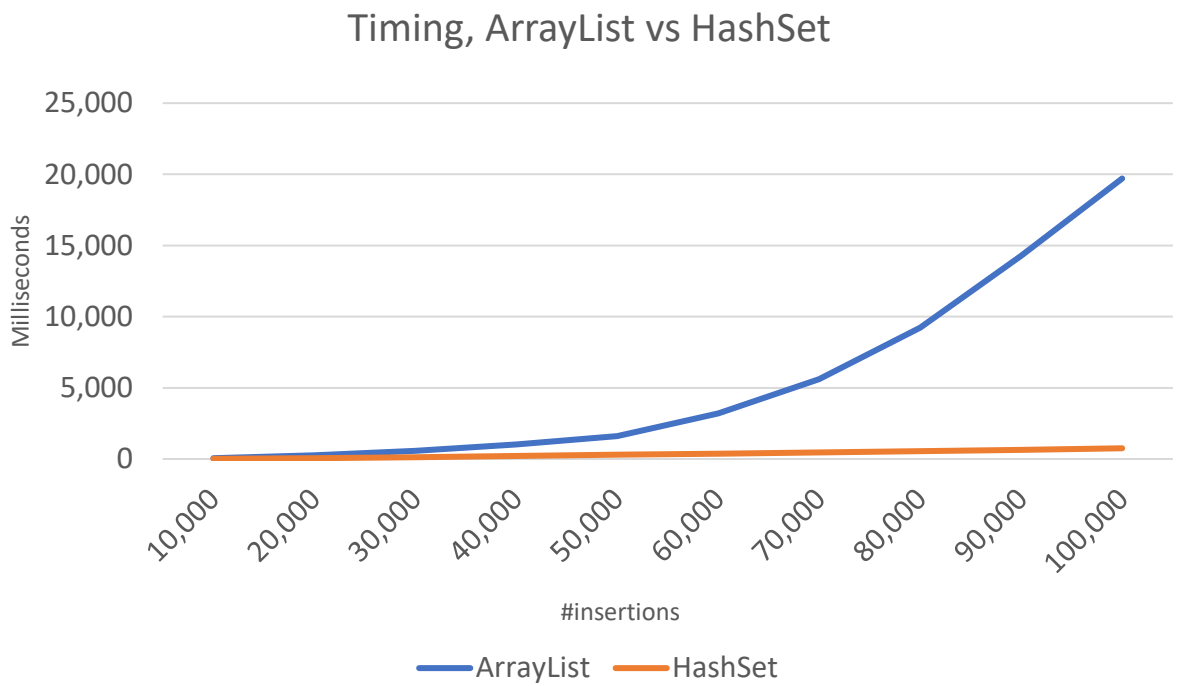
We define a simple hash function for a pair that just sums the hash values of its constituent fields.

Enumeration of rationals: Contrast performance of `ArrayList` and `HashSet`.

```
/* Output reduced fractions, i.e., positive rationals; no repeats. */
public static void timing() {
   HashSet<Rational> reduced = new HashSet();
   long startTime = System.currentTimeMillis();
   int rCount = 0;  // # of rationals so far.
   int d = 0;
   while ( rCount<100000 ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         /* Let z be the reduced form of the fraction (r+1)/(c+1). */
            Rational z = new Rational(r+1, c+1);
         if ( !reduced.contains(z) ) {
            /* System.out.println( z ); */
            reduced.add(z);
            rCount++;
            if ( rCount%10000==0 )
               System.out.println( System.currentTimeMillis()-startTime );
            }
            r--;
         }
         d++;
      }
   } /* timing */
```

Comment out the output statement so that it is not timed.
Then, time every 10,000 collection insertions.

Enumeration of rationals: performance of `ArrayList` vs. `HashSet`.



Performance of ArrayList is quadratic; performance of HashSet is linear.

Enumeration of rationals: On reflection, why are we bothering to maintain the collection of already-output rationals in the first place? Why not just output (r+1)/(c+1) when it is a reduced fraction?

The test for n/d being reduced is just gcd(n,d)==1?

☞ **Analyze first.**

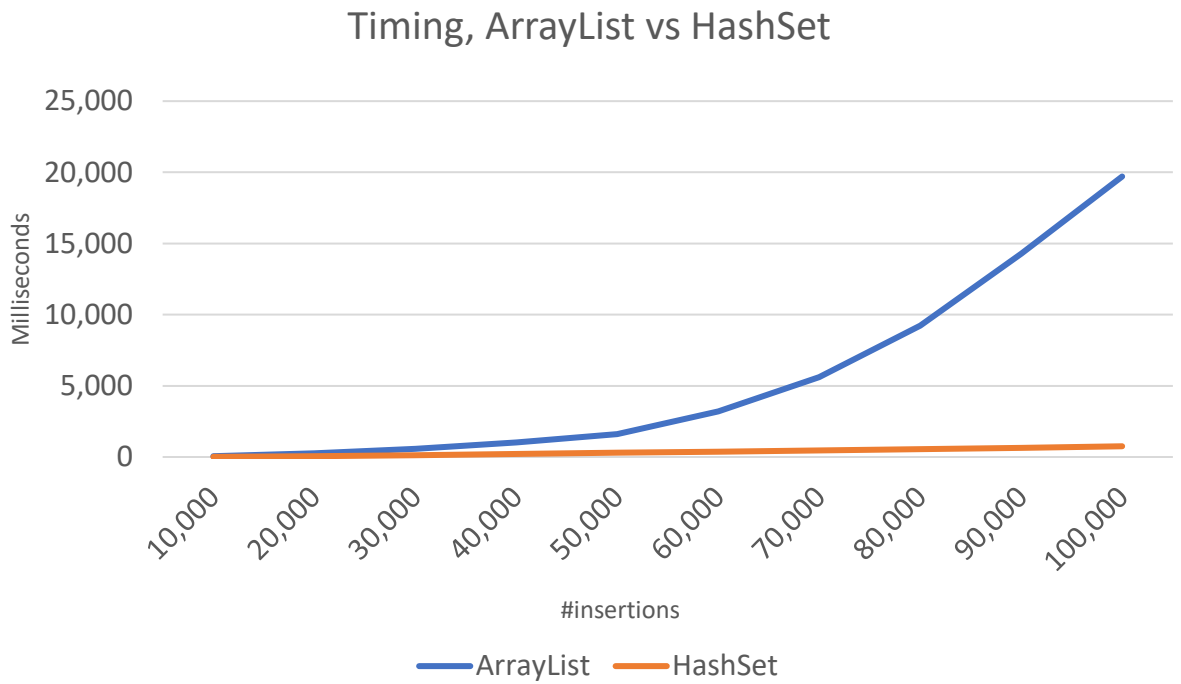
Enumeration of rationals: Contrast performance of `ArrayList`, `HashSet`, and gcd==1.

```
/* Output reduced fractions, i.e., positive rationals; no repeats. */
public static void timing() {

   long startTime = System.currentTimeMillis();
   int rCount = 0;  // # of rationals so far.
   int d = 0;
   while ( rCount<100000 ) {
      int r = d;
      for (int c=0; c<=d; c++) {
         if ( Rational.gcd(r+1,c+1)==1 ) {
            /* Let z be the reduced form of the fraction (r+1)/(c+1). */
               Rational z = new Rational(r+1, c+1);
            /* System.out.println( z ); */
            rCount++;
            if ( rCount%10000==0 )
               System.out.println( System.currentTimeMillis()-startTime );
            }
            r--;
         }
         d++;
      }
   } /* timing */
```

Only create the `Rational` when the fraction is in reduced form.

Enumeration of rationals: Contrast performance of `ArrayList`, `HashSet`, and gcd==1.



#insertions	ArrayList	HashSet	gcd
10,000	72	23	2
20,000	257	50	5
30,000	574	135	8
40,000	1035	220	11
50,000	1601	308	14
60,000	3206	372	16
70,000	5602	463	18
80,000	9236	550	20
90,000	14290	644	23
100,000	19711	750	27

Performance of ArrayList is quadratic, while performance of HashSet is linear. But in contrast, checking whether the fraction is in reduced form is practically instantaneous.

Enumerating a Collection: A small lose end.

Recall that one of the operations of a collection is to enumerate its elements. This is easy when we have direct access to the collection's implementation, e.g.,

```
/* Enumerate items of a collection implemented as a list ⟨A,size,n⟩. */
   for (int k=0; k<size; k++) /* Do whatever for A[k]. */
```

```
/* Enumerate items of a collection implemented as a histogram H[0..maxValue]. */
   for (int k=0; k<=maxValue; k++)
      for (int j=1; j<=H[k]; j++)
         /* Do whatever for k. */
```

But how can you enumerate the items of a collection when its implementation is hidden within a class? Specifically, how can your code be independent of the collection's implementation?

Enumerating a Collection: A small lose end.

Let `C<E>` be a generic subclass of `AbstractCollection<E>`. Let `c` be an object of an instantiation of `C<E>` for some specific element type `EL`. Then `c` is a collection of `EL` items, where the collection implementation is defined by `C<E>`.

An *iterator* for `c` is an object `i` that provides two methods:

- `i.hasNext()`, which returns a `boolean` that says whether the `i` can be pumped for yet another element of `c`.
- `i.next()`, which returns a value of type `EL`. Provided `i.hasNext()` has just returned **true**, invoking `i.next()` returns the "next" element of collection `c`, where the order of enumeration is beyond your control.

N.B. Although not technically accurate, you can think of there being a generic class `Iterator<E>` that has an instantiation `Iterator<EL>`, and `i` is an object of that class.

Enumerating a Collection: A small lose end.

The following code pattern can be used to pump collection c for elements until there are no more:

```
Iterator<EL> i = c.iterator();
while ( i.hasNext() ) {
    EL e = i.next();
    /* process element e. */
    }
```

Enumerating a Collection: A small lose end.

Suppose after having enumerated 100,000 rationals and storing them in `reduced`, you wanted to read them out from `reduced` and process them. Then you could do so with this instance of the code pattern above:

```
Iterator<Rational> i = reduced.iterator();
while ( i.hasNext() ) {
    Rational e = i.next();
    /* process element e. */
    }
```

This code would work regardless of whether reduced is implemented as an `ArrayList<Rational>` or a `HashSet<Rational>`. The details of how items are extracted from `reduced` are hidden in the implementation of the particular iterator `i` that is returned by `reduced.iterator()`.