CS 417 Prelim 2

Tuesday, April 1, 2003

Problem 1: 3D transformations (25 pts)

Write the 3x3 matrices for the following linear transformations:

1. (5 pts) A rotation by 30 degrees around the x axis.
2. (7 pts) A rotation by 30 degrees around the vector $(1, 1, 1)$. It's fine to leave this as a product of matrices, but the entries in the matrices should be numbers (square roots are fine). Your solution should not have matrix inversions in it.

   Hint: You do not need to compute inverse trigonometric functions for this problem.

   Hint: $\sin\theta = \sqrt{1 - \cos^2\theta}$.

Write the 4x4 matrix for this affine transformation:

3. (7 pts) A viewing transformation for a camera at the position $(0, 3, 4)$ looking at the origin with up vector $(0, 1, 0)$. It is OK to use a matrix inverse in your answer.

Write the 3x4 matrix for this projective transformation:

4. (6 pts) A projection transformation for a perspective camera with an aspect ratio (width:height) of 1.5 and vertical field of view of 60°. The view frustum of the camera in canonical position should map to the range $[-1, 1]$ in x and y.

For reference: $\cos 30° = \frac{\sqrt{3}}{2}$; $\sin 30° = \frac{1}{2}$; $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$. The canonical camera position is at the origin looking in the $-z$ direction with $+y$ up.

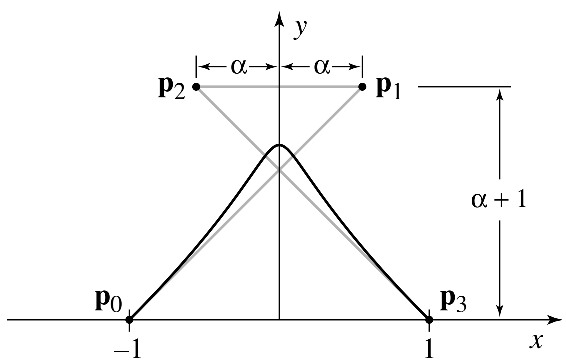
Problem 2: Splines (30 Pts)

1. (10 pts) Write out a matrix that looks like this and fill in yes or no in each space to indicate which type of spline has which properties.

	C^1	C^2	stays in convex hull of control points	interpolates control points
Hermite				
Cubic Bézier				
Quadratic B-Spline				
Cubic B-Spline				
Catmull-Rom				

Assume Hermite and Bézier segments are joined in the usual way. In interpreting the column headers for Hermite splines, the tangents do not count as "control points"—only the points.

2. (20 pts) Consider a cubic Bézier segment with control points at $(-1, 0)$, $(\alpha, \alpha + 1)$, $(-\alpha, \alpha + 1)$, and $(1, 0)$:



At one particular value of α, this curve will form a cusp, or a sharp point. Call this value α_c.

(a) Sketch the shape of the spline curve for α just below, at, and just above α_c.

(b) For $\alpha = \alpha_c$, what are the parametric and geometric continuities of the spline at the cusp?

(c) What is the value of t at the cusp? Give a very brief argument to support your answer.

(d) Plot the spline's two scalar coordinate functions $x(t)$ and $y(t)$ for $\alpha = \alpha_c$. How can you tell from loooking at these graphs that there is a cusp?

(e) What is the value of α_c? Show your derivation.

Problem 3: Perspective (20 pts)

1. (4 pts) If I take a picture of a basketball from a distance of 1 meter, then carry my camera to a point 10 meters away and take another picture of the ball, will the two images see the same fraction of the surface? If not, which one sees more?

One could imagine producing the following image in a number of ways:

i. By taking a photo of a very small man standing on someone's hand.

ii. By taking a photo of a normal-size man standing near the camera, scaling it down, and compositing it into a different photo (taken using the same camera and lens) of someone holding out his hand.

iii. By taking a photo of two normal-size people, one near the camera and one far away, while paying careful attention to the positioning of the near person's hand.

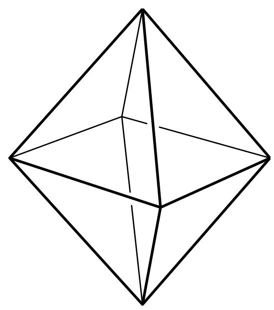
The image above is 450 pixels high, and the two heads measure 90 and 15 pixels high. Assume all heads are 30 cm high.

2. (8 pts) Paying attention only to perspective effects, (that is, assuming compositing is artifact-free) which of these three images (i, ii, or iii) ends up looking different from the other two? Briefly explain.
3. (8 pts) The camera has an image height of 24 mm. Assuming case (iii) above, how far away are the two subjects if we use a wide-angle lens of focal length 24 mm? A normal lens of focal length 48 mm? A telephoto lens of focal length 96 mm?

(Photo courtesy of Seth Teller, who says, "no computers were used to make this picture.")

Problem 4: Triangle meshes (25 pts)

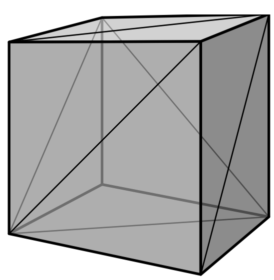
An octahedron is a surface with 6 vertices and 8 equilateral triangles:



1. (5 pts) Write out an indexed triangle set representation of an octahedron. Position it with the vertices sitting on the coordinate axes and all 1 unit from the origin. Make sure your triangles are oriented consistently.

2. (7 pts) Give a sequence of edge collapses that will make the octahedron into a tetrahedron. Each edge should be specified by the indices of its endpoints in your answer to the previous part.

Consider the following triangulated cube:



3. (7 pts) Which of the face diagonals are safe for edge collapses and which are not? Refer to the faces as top, bottom, left, right, front, back.

4. (6 pts) What is the smallest change to the mesh (without changing the cube geometry) that will make all edges safe to collapse?