CASE $e = e_1\ e_2$ $\qquad \vdash e_1\ e_2 : \tau$

BY INVERSION,

$\vdash e_1 : \tau' \to \tau$

$\vdash e_2 : \tau'$

BY IHOP,

$e_1 \to^* v_1$

$e_2 \to^* v_2$

BY CANONICAL FORMS,

$v_1 = \lambda x : \tau' .\ e'$

$$e \to^* v_1\ v_2 \to \underbrace{e' \{ v_2 / x \}}_{\text{DARK IHOP}} \to^* v_3$$

INDUCT ON e.

CASE x $\quad \Gamma \vdash x : \tau$

INVERSION $\Rightarrow \Gamma(x) = \tau$

$x = x_i$

$\tau = \tau_i$

$e\{v_1/x\} \cdots \{v_k/x_k\} = v_i$

$R_{\tau_i}(v_i)$

CASE ()

$\tau = \text{unit}$

$e\{\cdots\} = ()$

$R_{\text{unit}}(())$

CASE $e = \lambda x : \tau' . e'$

BY INVERSION,

$\tau = \tau' \to \tau''$

$\Gamma, x:\tau' \vdash e' : \tau''$

$e\{ \qquad\qquad\qquad \}$

LET e'' BE SOME EXPR.

S.T. $R_{\tau'}(e'')$

- $\cdot \vdash e'' : \tau'$
- e'' HALTS

$\exists v'' \quad e'' \longrightarrow^* v''$

THEREFORE,

$R_{\tau'}(v'')$

IHOP TO e'

$R_{\tau''}(e'\{v_1/x_1\} \cdots \{v_k/x_k\}\{v''/x\})$

$R_{\tau''}(e\ e'')$

$$R_{\tau' \to \tau''}\left(e \{ \cdot \cdots \} \right)$$