

## PRESERVATION

$\vdash e : \tau$  AND  $e \rightarrow e' \Rightarrow \vdash e' : \tau$

INDUCT ON  $e \rightarrow e'$ .

### ADD

$$e = n_1 + n_2 \quad e' = n$$

WHERE  $n = n_1 + n_2$

$$\frac{}{\vdash n : \text{int}} \text{T-INT}$$

### $\beta$ -REDUCTION

$$e = (\lambda x : \tau'. e_1) v$$

$$e' = e_1 \{v/x\}$$

$$\frac{}{\vdash e : \tau} \text{T-APP}$$

$\exists$  DERIVATIONS FOR

$$\vdash \lambda x : \tau'. e_1 : \tau' \rightarrow \tau$$

$$\begin{array}{c} \text{AND } \vdash v : \tau' \\ \Delta \quad \frac{x : \tau' \vdash e_1 : \tau}{\text{T-ABS}} \end{array}$$

$$e' = e_1 \{v/x\}$$

$$\vdash e' : \tau$$

CONTEXT

$$\exists E \quad \begin{array}{l} e = E[e_1] \\ e' = E[e_2] \end{array}$$

WHERE  $e_1 \rightarrow e_2$   
 (BY A SMALL INDUCTION ON EVAL CONTEXTS)

$$\vdash e_1 : \tau_1$$

BY INDUCTION HYP.,

$$\vdash e_2 : \tau_1$$

BY CONTEXT LEMMA,

$$\vdash E[e_2] : \tau$$

# PROGRESS

$\vdash e:\tau$   $\Rightarrow$   $e$  IS A VALUE OR  $e \rightarrow e'$

INDUCT ON  $\vdash e:\tau$ .

T-VAR

IMPOSSIBLE

T-UNIT, T-INT, T-ABS

$e$  IS A VALUE

T-ADD

$$e = e_1 + e_2$$

AND  $\vdash e_1 : \text{int}$

AND  $\vdash e_2 : \text{int}$

•  $e_1$  IS NOT A VALUE

$$e_1 \rightarrow e_1'$$

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$$e_1 + e_2 \rightarrow e_1' + e_2 \quad \text{CONTEXT}$$

- o o.w.,  $e_1$  IS A VALUE.  
IF  $e_2$  IS NOT A VALUE

$$\frac{}{e_1 + e_2 \rightarrow e_1 + e_2'} \text{ CONTEXT}$$

- o BOTH VALUES

$$e_1 = v_1 \quad e_2 = v_2$$

$$\frac{}{e_1 + e_2 \rightarrow v} \text{ ADD}$$

## T-APP

$$e = e_1 \ e_2$$

$$\text{AND } \vdash e_1 : \tau' \rightarrow \tau$$

$$\text{AND } \vdash e_2 : \tau'$$

- o ASSUME  $e_1$  IS NOT A VALUE
- o o.w.,  $e_1$  IS A VALUE.  
ASSUME  $e_2$  IS NOT.

◦  $e_1$  AND  $e_2$  BOTH VALUES.

$$e_1 = \lambda \dots$$

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$$e_1, e_2 \rightarrow e^{\{e_2/x\}} \quad (3)$$