

★ TYPES ★

for the λ -calculus

ANNOUNCEMENTS

- HW #5 OUT TODAY
AUTOMATICALLY VERIFY IMP PROGRAMS
USING HOARE LOGIC PROOFS
 - YOU CAN NOW CHECK YOUR SLIP DAYS
ON CMS (IN A FAKE "ASSIGNMENT")
-

TYPES!

A TYPE IS:

- A SET OF VALUES
 - A STATIC OVERAPPROXIMATION
OF DYNAMIC BEHAVIOR
 - A LIGHTWEIGHT FORMAL METHOD
FOR REASONING ABOUT PROGRAMS
-

THE SIMPLY-TYPED λ -CALCULUS

— SYNTAX —

$$e ::= x$$
$$| \lambda x:\tau. e$$
$$| e_1 e_2$$
$$| n$$
$$| e_1 + e_2$$
$$| () \leftarrow \text{"UNIT"}$$
$$v ::= \lambda x:\tau. e \mid n \mid ()$$
$$\tau ::= \text{int} \mid \text{unit} \mid \tau_1 \rightarrow \tau_2$$

— SEMANTICS —

EXACTLY THE SAME.
JUST IGNORE THE TYPES!

— BUT WHY? —

WE GET A GUARANTEE:

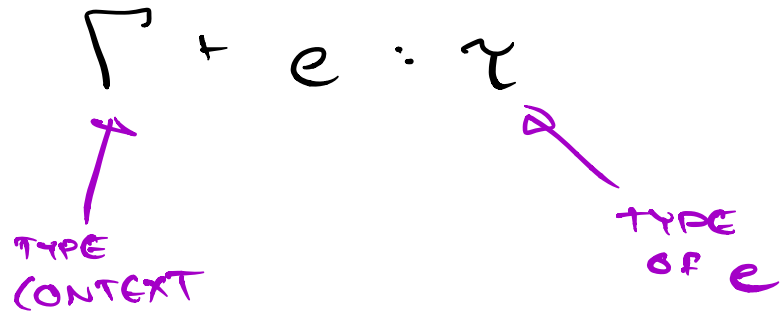
WELL-TYPED PROGRAMS
WILL NOT GET STUCK.

$42 + \lambda x: \text{int}. x$



THE TYPING RELATION

A.K.A. "JUDGMENT"



" e HAS TYPE τ IN CONTEXT Γ "

Γ PARTIAL FUNCTION FROM VARIABLES TO τ

$\Gamma [x \mapsto \tau]$ = Γ WITH A NEW TYPE PAIR

$\Gamma, x : \tau$

"WELL-TYPED IN Γ "

"WELL-TYPED"

$\exists \tau. \Gamma \vdash e : \tau$ $\Gamma = \emptyset$

$\exists \tau. \vdash e : \tau$

$\lambda x : \text{int}. (\lambda y : \text{int}. (x + y))$

TYPING RULES

$$\frac{}{\Gamma \vdash n : \text{int}} \text{T-INT}$$

$$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}} \text{T-ADD}$$

$$\frac{}{\Gamma \vdash () : \text{unit}} \text{T-UNIT}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \text{T-VAR}$$

$$\frac{\Gamma[x \mapsto e] \vdash e : \tau'}{\Gamma \vdash (\lambda x : \tau. e) : \tau \rightarrow \tau'} \text{T-ABS}$$

$$\frac{\Gamma + e_1 : \tau \rightarrow \tau' \quad \Gamma + e_2 : \tau}{\Gamma + e_1, e_2 : \tau'} \quad \tau\text{-APP}$$

Let's Type A Program!

$e = (\lambda x: \text{int}. x + 40) \ 2$

$\Gamma \vdash e : \text{int}$

$$\begin{array}{c} \Gamma(x) = \text{int} \\ \hline \frac{}{(x: \text{int}) \vdash x : \text{int}} \text{T-VAR} \quad \frac{}{(x: \text{int}) \vdash 40 : \text{int}} \text{T-INT} \\ \hline \frac{}{(x: \text{int}) \vdash x + 40 : \text{int}} \text{T-ADD} \\ \hline \frac{}{\vdash (\lambda x: \text{int}. x + 40) : \text{int} \rightarrow \text{int}} \text{T-LAMBDA} \quad \frac{}{\vdash 2 : \text{int}} \text{T-INT} \\ \hline \frac{}{\vdash e : \text{int}} \text{T-APP} \end{array}$$

NEXT TIME : PROVING TYPE SAFETY

IF $\vdash e : \tau$

AND $e \rightarrow^* e'$

AND $e' \neq \text{value}$ ($\exists e'' e \rightarrow e''$)

THEN e' IS A VALUE

AND $\vdash e' : \tau$.

EVEN MORE
ANNOUNCEMENTS

- PROOF-WRITING WORKSHOP!
PROBABLY TUESDAY THE 18TH @ 7pm
- MID-SEMESTER COURSE FEEDBACK!
LOOK FOR THE SURVEY LINK TODAY