CS 4110 · LECTURE 18

TYPES for the λ-calculus

ANNOUNCEMENTS

- HW #5 OUT TODAY

  AUTOMATICALLY VERIFY IMP PROGRAMS USING HOARE LOGIC PROOFS

- YOU CAN NOW CHECK YOUR SLIP DAYS ON CMS (IN A FAKE "ASSIGNMENT")

TYPES!

A TYPE IS:

- A SET OF VALUES
- A STATIC OVERAPPROXIMATION OF DYNAMIC BEHAVIOR
- A LIGHTWEIGHT FORMAL METHOD FOR REASONING ABOUT PROGRAMS

The Simply-Typed λ-Calculus

— Syntax —

$$
\begin{aligned}
e ::= \; & x \\
\mid \; & \lambda x{:}\tau.\, e \\
\mid \; & e_1 \; e_2 \\
\mid \; & n \\
\mid \; & e_1 + e_2 \\
\mid \; & () \quad \leftarrow \text{"unit"}
\end{aligned}
$$

$$
\begin{aligned}
v &::= \lambda x{:}\tau.\, e \mid n \mid () \\
\tau &::= \text{int} \mid \text{unit} \mid \tau_1 \to \tau_2
\end{aligned}
$$

— Semantics —

Exactly the same.

Just ignore the types!

But why?

We get a guarantee:

Well-typed programs will not get stuck.

$42 + \lambda x : \text{int} . x\ ()\ 7$

The Typing Relation

A.K.A. "judgment"

$$\Gamma \vdash e : \tau$$

- Γ: type context
- τ: type of e

"e has type τ in context Γ"

Γ partial function from variables to τ

$\Gamma[x \mapsto \tau]$ = Γ with a new type pair

$\Gamma, x : \tau$

"well-typed in Γ": $\exists \tau. \Gamma \vdash e : \tau$

"well-typed": $\Gamma = \emptyset$, $\exists \tau. \vdash e : \tau$

$\lambda x : \text{int}. (\lambda y : \text{int}. (x + y))$

Typing Rules

$$\frac{}{\Gamma \vdash n : \text{int}}\ \text{T-INT}$$

$$\frac{\Gamma \vdash e_1 : \text{int} \qquad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}\ \text{T-ADD}$$

$$\frac{}{\Gamma \vdash () : \text{unit}}\ \text{T-UNIT}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}\ \text{T-VAR}$$

$$\frac{\Gamma[x \mapsto \tau],\ x : \tau \vdash e : \tau'}{\Gamma \vdash (\lambda x : \tau.\ e) : \tau \to \tau'}\ \text{T-ABS}$$

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'}\ \text{T-APP}$$

Let's Type a Program!

$e = (\lambda x{:}\mathrm{int}.\ x + 40)\ 2$

$\Gamma \vdash e : \mathrm{int}$

$$\dfrac{\dfrac{\dfrac{\Gamma(x) = \mathrm{int}}{(x:\mathrm{int}) \vdash x : \mathrm{int}}\ \text{T-VAR} \qquad \dfrac{}{(x:\mathrm{int}) \vdash 40 : \mathrm{int}}\ \text{T-INT}}{(x:\mathrm{int}) \vdash x + 40 : \mathrm{int}}\ \text{T-ADD}}{\vdash (\lambda x{:}\mathrm{int}.\ x+40) : \mathrm{int} \to \mathrm{int}}\ \text{T-ABS} \qquad \dfrac{}{\vdash 2 : \mathrm{int}}\ \text{T-INT}$$

$$\vdash e : \mathrm{int}\ \text{T-APP}$$

NEXT TIME: PROVING TYPE SAFETY

IF $\vdash e : \tau$

AND $e \longrightarrow^* e'$

AND $e' \not\longrightarrow$ $\quad (\nexists e''\ e \longrightarrow e'')$

THEN e' IS A VALUE

AND $\vdash e' : \tau$.

EVEN MORE ANNOUNCEMENTS

- PROOF-WRITING WORKSHOP!
  PROBABLY TUESDAY THE 18TH @ 7pm
- MID-SEMESTER COURSE FEEDBACK!
  LOOK FOR THE SURVEY LINK TODAY