$\{ x = n \land n > 0 \}$ $= P$

```
y := 1;                 ← c1
while x > 0 do (
    y := y × x;         } c2        } c
    x := x − 1
)
```

$\{ y = n! \}$ $= Q$

TINY EXTENSION TO THE ASSERTION LANGUAGE

$c = c_1 ; c_2$

$$\text{SEQ}\quad \frac{\vdash \{P\}\ c_1\ \{R\} \qquad \vdash \{R\}\ c_2\ \{Q\}}{\vdash \{P\}\ c\ \{Q\}}$$

$$I = y = n \times (n-1) \times (n-2) \times \cdots \times (x+1)$$

$$= (x! \, y = n!) \land x \geq 0$$

$c_1 = y := 1$ $\qquad$ $c_2 =$ while...

$$\text{ASSIGN}\ \frac{}{\{P\}\ c_1\ \{I\}}$$

$$P = I[1/y]$$
$$= (x! \times 1 = n!) \wedge (x \geq 0)$$

$$P = x = n \wedge n > 0$$
$$\Rightarrow (x! \times 1 = n!) \wedge x \geq 0$$

$I[1/y]$ ↗

$$\text{CONSEQUENCE}\ \frac{}{\{P\}\ c_1\ \{I\}}$$

It remains to show:

$$\{I\}\ c_2\ \{Q\}$$

$\{I \wedge b\}$ $\quad$ $\{P\}$ $\quad$ $\{P\} \wedge$ $\quad$ $\{I\}$

$$\text{SEQ}\ \frac{\{I\}\ C_{B1}\ \{K\} \quad \{K\}\ C_{B2}\ \{I\}}{\{I \wedge b\}\ C_{B1};C_{B2}\ \{I\}}$$

$C_{B1} = y := y * x$

$C_{B2} = x := x - 1$

$R = ((x-1)! * y = n!) \wedge (x-1) \geq 0$

$R' = ((x-1)! * y * x = n!) \wedge (x-1) \geq 0$

$I \wedge x > 0$

$\Rightarrow R'$

$$\text{WHILE}\ \frac{\vdash \{I \wedge x > 0\}\ C_{B1};C_{B2}\ \{I\}}{\vdash \{I\}\ C_2\ \{I \wedge x \leq 0\}}$$

$I \wedge x \leq 0 \Rightarrow y = n!$

$$\text{CONSEQUENCE}\ \frac{}{\{P\}\ c\ \{Q\}}$$