

$$X = \bigcup_i F^i(\emptyset)$$

1. X IS SOME FIXED PT OF F

$$F(X) = F\left(\bigcup_i F^i(\emptyset)\right) \dots X$$

$$= \bigcup_i F(F^i(\emptyset))$$

$$= \bigcup_i F^{i+1}(\emptyset)$$

$$= \emptyset \cup \bigcup_i F^{i+1}(\emptyset)$$

$$= F^0(\emptyset) \cup \bigcup_i F^{i+1}(\emptyset)$$

$$= \bigcup_i F^i(\emptyset)$$

$$= X$$

2. $\exists Y \subset X$

ALSO A FIXED PT
ASSUME BY N.O.C. THAT $Y \subset X$ IS A F.P.

$$F^i(\emptyset) \subseteq Y \quad \forall i$$

BASE CASE $i=0$ $F^0(\emptyset) = \emptyset \subseteq Y$

INDUCTIVE CASE

ASSUME $F^i(\emptyset) \subseteq Y$

$$\Rightarrow F^{i+1}(\phi) \subseteq Y$$

$$\boxed{X \subseteq Y \Rightarrow F(X) \subseteq F(Y)}$$

$$F(F^i(\phi)) \subseteq F(Y)$$

$$F^{i+1}(\phi) \subseteq F(Y)$$

$$F^{i+1}(\phi) \subseteq Y$$

$$\underbrace{F^0(\phi) \subseteq F^1(\phi) \subseteq \dots}_{\subseteq Y} \cup_i F^i(\phi) \subseteq Y$$

$$X \subseteq Y$$

$$\Rightarrow \Leftarrow$$

$$\begin{aligned}
&= \{ (\sigma, \sigma'') \mid \\
&\quad (\sigma, \sigma'') \in \llbracket c \rrbracket \\
&= \{ (\sigma, \sigma'') \mid \exists \sigma' \\
&\quad (\sigma, \sigma') \in \llbracket c \rrbracket \wedge \\
&\quad (\sigma', \sigma'') \in \llbracket \text{skip} \rrbracket \} \\
&= \llbracket c; \text{skip} \rrbracket
\end{aligned}$$