HW 2

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Entrywise errors Suppose $Ax = b$ and $\hat{x} = x + e$ satisfies

$$A\hat{x} = b + r.$$

Suppose we are given an elementwise relative residual bound $\max_i |r_i|/|b_i| < \gamma$; then show that

$$\max_i |e_i|/|x_i| \leq \gamma \| \operatorname{diag}(x)^{-1} A^{-1} \operatorname{diag}(b) \|_\infty.$$

2: Delicious differentials Suppose $PA = LU$, and assume that there are no ties during partial pivoting. Write a short MATLAB routine to compute the variations δL and δU for a given change δA

```
function [dL, dU] = hw2deriv(L, U, P, dA)
```

3: Complementary conditioning Suppose A is symmetric and positive definite. Show that if S is a Schur complement that appears during factorization of A, then $\kappa_2(S) \leq \kappa_2(A)$. *Hint:* What is the relation between $\kappa_2(A)$ and $\kappa_2(L)$ where $A = LL^T$ is a Cholesky factorization? What is the relation between the Cholesky factorization of A and of S?

4: Run for the border Suppose A satisfies

$$\sigma_1(A) \leq C\sigma_{n-1}(A)$$

for some modest C, but $\sigma_n(A)$ is very close to zero, and let `solveA` be a backward stable function for solving a linear system involving the matrix A. Assuming the bordered system

$$\begin{bmatrix} A & b \\ c^T & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$$

is well conditioned. Write a MATLAB code that uses block Gaussian elimination based with one step of iterative refinement to solve the system:

```
function [x, y] = hw2refine(solveA, b, c, d, f, g)
```

Argue that this computation is backward stable.