Midterm

Due: Monday, Oct 28.

Choose any five problems. You may use the textbook or other references (with citation). You may discuss problems with me, but not with each other. **Test your codes**, and document your tests. There will be no credit for code that is clearly broken and untested. There will also be points deducted if you use `inv` or equivalent constructs like `A\eye(size(A))`.

Bonus point (not one of the five questions): Tell me something you particularly like about the class or give me a suggestion to improve it.

1. Consider the function $f(x) = \sqrt{1+x} - \sqrt{1-x}$ defined on $[-1, 1]$. Give an example that illustrates that the following naive MATLAB algorithm can have poor relative accuracy in floating point:

```
function f = mtbadf(x)
  f = sqrt(1+x)-sqrt(1-x);
```

   Write an alternate formulation `mtgoodf` that has good relative accuracy (relative error of a few machine epsilon) over the entire range $[-1, 1]$.

2. Suppose $PM = LU$ for a square matrix $M \in \mathbb{R}^{n\times n}$, and let $f(s)$ be the $(1, 1)$ entry for $(M + sE)^{-1}$, where $E \in \mathbb{R}^{n\times n}$ is a given fixed matrix. following routine to compute $f'(0)$. Your code should take $O(n^2)$ time.

```
function [dfds] = mtderiv(L,U,P,E)
```

3. Suppose A is symmetric positive definite and the Cholesky factorization $A = R^T R$ is given. Write a function to compute an LU factorization of A in $O(n^2)$ time.

```
function [L,U] = mtcholtolu(R)
```

4. Suppose $\hat{A} \in \mathbb{R}^{n\times n}$, and let $C \in \mathbb{R}^{n\times k}$ with $k < n$. Assuming that C is not rank deficient, find $B \in \mathbb{R}^{k\times n}$ to minimize $\|A - CB\|_F$.

```
function [B] = mtfindB(A,C)
```

5. Let (x_i, y_i) for $i = 1, \ldots, n$ be the coordinates of n points in the plane. Let $D \in \mathbb{R}^{n \times n}$ be a matrix of squared Euclidean distances, i.e. $d_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2$. Supposing the first three points are not co-linear, write a function to recover all the points from the distance matrix and the coordinates of these two points.

   ```
   function [x,y] = mtrecover(D, x1,y1, x2,y2, x3,y3)
   ```

6. Suppose L is nonsingular and lower bidiagonal (i.e. l_{ij} is only nonzero for $i = j$ or $i = j + 1$). Write a code to compute $\kappa_\infty(L)$ in $O(n)$ time.

   ```
   function [condL] = mtcondL(L)
   ```

   Note: The MATLAB `cond` function is not $O(n)$!