Final

You may use any standard reference (book, notes, HW solutions, MATLAB help), but please do not consult with classmates or others apart from me. You may use any MATLAB functions you wish in your solutions, and you *should* use MATLAB (or Octave) to check correctness of your codes. You will be graded on the correctness (and correct efficiency) of your solutions, but include derivations, test cases, etc. that you think will help me evaluate your solutions. Submit via CMS by 11:59 pm on Dec 16.

1. Suppose A is symmetric and positive definite, and let L be the lower triangular Cholesky factor. Give *short* arguments for each of the following:

1. For all i, $l_{ii}^2 \leq a_{ii}$. If equality holds for all i, then A is diagonal.
2. The columns of L are orthonormal with respect to the inner product induced by A^{-1}.
3. If A has the block 2-by-2 form
$$A = \begin{bmatrix} A_{11} & A_{21}^T \\ A_{21} & A_{22} \end{bmatrix}$$
where A_{11} and A_{22} are square and L is similarly partitioned, then L_{21} has the same rank as A_{21}.

2. Suppose $A \in \mathbb{R}^{m \times n}$ with $m > n$, and $b \in \mathbb{R}^m$, and consider the iteration
$$u^{(k+1)} = u^{(k)} + \alpha A^T(b - Au^{(k)}).$$

1. What is the fixed point?
2. What is the iteration matrix?
3. What is the spectral radius of the iteration matrix?
4. For what range of positive α values will the iteration converge?

3. Suppose $A \in \mathbb{R}^{n \times m}$ is a rectangular matrix, and let $\mathcal{I}$ and $\mathcal{J}$ be row and column index sets. Write a routine to optimally approximate A in terms of these rows and columns; that is, find B to minimize $\|A - \hat{A}\|_F^2$ where

$$\hat{A} = (A_{:,\mathcal{J}})B(A_{\mathcal{I},:}).$$

Your code should have the form

```
function [B] = p3minimize(A, I, J)
```

4. Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric and nonsingular, and $A = LU$ where L is unit lower triangular and U is upper triangular. Write a routine that, given U, returns the number of positive eigenvalues of A:

```
function [num_pos] = p4count(U)
```

5. Consider the block 2-by-2 matrix

$$T = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$$

where A and B are square. Assuming A and B have no common eigenvalues, find R such that

$$S^{-1}TS = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}, \quad S = \begin{bmatrix} I & R \\ 0 & I \end{bmatrix}.$$

Your routine should have the form

```
function [R] = p5blockdiag(A, B, C)
```