Practice midterm

Due: Never!

For the midterm, you are allowed to use texts, papers, or other references (with citation). You should not ask help from any other person, whether inside or outside the class. You should not worry if you do not get all the answers; this is a sign that the test is doing a proper job! We reserve the right to ask follow-up questions in person (e.g. to determine whether it makes sense to assign partial credit). You may ask us to clarify ambiguities or perceived errors in the prompt, but please do not ask for hints.

For the practice, of course, you can ask whatever you want! Bring questions to Friday lecture if there are problems you consider particularly vexing.

1. Given the described factorization, write a short *efficient* MATLAB expression to evaluate each of the following
   (a) Given `[L,U,P] = lu(A)`, solve $Ax = b$.
   (b) Given `[U,S,V] = svd(A,0); sigma = diag(S);` minimize $\|Ax - b\|^2 + \lambda\|x\|^2$.
   (c) Given a `[Q,R] = qr(A)` where A is square and nonsingular, find the jth diagonal element of A^{-1}.

2. Write a line of MATLAB to compute $y = \cos(x) - 1$ accurately in floating point for $x \in [-0.5, 0.5]$. Your code should not lose accuracy for $|x| \ll 1$.

3. In class, we sketched a factorization based approach to solving the least squares problem of minimizing $\|Ax - b\|_M^2$. Given the required factorizations, give a simple expression for the derivative of x with respect to a given small perturbations in M (call it `dM` in MATLAB).

4. Rewrite the following function to run in $O(n)$ time:

```
function [y] = triuLR(u, v, x)
y = triu(u*v')*x;
```

5. Suppose T is a symmetric tridiagonal matrix with diagonal entries $\alpha_1, \ldots, \alpha_n$ and off-diagonal entries $\beta_1, \ldots, \beta_{n-1}$. Without using MATLAB sparse matrices, write an $O(n)$ code that works directly with the coefficient vectors to check whether T is positive definite:

```
function [p] = check_pd(alpha, beta)
% Return p = 0 if the T is positive definite, p > 0 otherwise
```

6. Let $\alpha \in \mathbb{R}^+$ and $u \in \mathbb{R}^n$ be given. Rewrite the following MATLAB function to run in $O(n)$ time

```
function d = choldiag(alpha, u)
n = length(u);
d = diag(chol(eye(n) + alpha*u*u'));
```