HW 3
Due by lecture on Mon, Feb 13

Remember that you may (and should!) talk about the problems amongst yourselves, or discuss them with me or the TA, providing attribution for any good ideas you might get – but your final write-up should be your own.

1: Basics

1. Write all the solutions to the equation

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

2. Try solving the above system in MATLAB:

```
A = [1, 2; 2, 4];
b = [3; 6];
v = A\b;
```

   What messages do you see? Why?

2: SPD matrix properties Suppose A is symmetric and positive definite.

1. Show that the diagonal elements of A are all positive.

2. For what values of a is the matrix

$$M(a) = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix}$$

   positive definite? Note: a symmetric matrix is positive definite iff all its eigenvalues are positive.

3: A little calculus Suppose $A(t)$ is a smoothly-varying matrix-valued function of t. Use implicit differentiation to find a formula for

$$B(t) = \frac{d}{dt}\left(A^{-1}(t)\right),$$

assuming that the inverse is well defined.

4: Efficient MATLAB Suppose $x, y, z \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$. Write efficient code to compute each of the following expressions. You should not use `inv` to compute explicit inverses.

1. $x^T A^{-1} y$
2. The arithmetic mean of the entries of A^{-1}.
3. $BABz$ where $B = xy^T$.