# Practice prelim 1

Remember: all three exams are closed book, but you may bring one letter-sized piece of paper with writing on both sides. The actual exam will be one hour.

1. True or false:

   (a) If an algorithm produces a solution with small backward error and the condition number is small, then the forward error is also small.

   (b) Floating point multiplication and addition are distributive, i.e. `x*(y+z)` and `x*y+x*z` always produce exactly the same result.

   (c) IEEE arithmetic obeys the bound $\mathrm{fl}(x \times y) = (x \times y)(1+\delta)$, $|\delta| < \epsilon_{\mathrm{mach}}$, for every possible floating point number $x$ and $y$.

   (d) If $\det(A)$ is close to zero, then $A$ is very nearly singular.

   (e) It is always true that $\|A\|_1 = \|A^T\|_\infty$

2. A program to evaluate $f(x)$ returns an approximate value $\hat{f}(x)$. Suppose $\hat{f}(x) = f(x(1+\delta))$ and $f'(x) = 1$. Write an expression for the (approximate) relative error in the computed $f$.

3. For $x > 1$, we can solve the equation $\cosh(y) = x$ for $x$ with the formula:

$$y = -\ln\left(x - \sqrt{x^2 - 1}\right).$$

   We implement this formula in the following MATLAB function:

```
function y = myarccosh(x)
  y = log( x-sqrt(x^2-1) );
```

   What is the output of the function for $x = 10^9$? Suggest an alternative organization that is more accurate for large $x$.

4. Compute the factorization $A = LU$ (no pivoting) for

$$A = \begin{bmatrix} 4 & 9 \\ 12 & 34 \end{bmatrix}.$$

5. Suppose $A$ is a nonsingular matrix, and we have computed $PA = LU$. Without using $A$ directly or writing explicit inverses (e.g. `inv(L)` or `U^(-1)`), write a MATLAB function that returns the $(1,1)$ entry of $A^{-1}$.