1. Let \( d \) be any positive integer not equal to 2, 5 or 13. Show that one can find distinct \( a, b \) in the set \( \{2, 5, 13, d\} \) such that \( ab - 1 \) is not a perfect square.

2. Given a point \( P_0 \) in the plane of the triangle \( A_1A_2A_3 \). Define \( A_s = A_{s-3} \) for all \( s \geq 4 \). Construct a set of points \( P_1, P_2, P_3, \ldots \) such that \( P_{k+1} \) is the image of \( P_k \) under a rotation center \( A_{k+1} \) through an angle 120 clockwise for \( k = 0, 1, 2, \ldots \). Prove that if \( P_{1986} = P_0 \), then the triangle \( A_1A_2A_3 \) is equilateral.

3. To each vertex of a regular pentagon an integer is assigned, so that the sum of all five numbers is positive. If three consecutive vertices are assigned the numbers \( x, y, z \) respectively, and \( y < 0 \), then the following operation is allowed: \( x, y, z \) are replaced by \( x + y, -y, z + y \) respectively. Such an operation is performed repeatedly as long as at least one of the five numbers is negative. Determine whether this procedure necessarily comes to an end after a finite number of steps.

4. Let \( A, B \) be adjacent vertices of a regular \( n \)-gon \( (n \geq 5) \) with center \( O \). A triangle \( XYZ \), which is congruent to and initially coincides with \( OAB \), moves in the plane in such a way that \( Y \) and \( Z \) each trace out the whole boundary of the polygon, with \( X \) remaining inside the polygon. Find the locus of \( X \).

5. Find all functions \( f \) defined on the non-negative reals and taking non-negative real values such that: \( f(2) = 0, f(x) \neq 0 \) for \( 0 \leq x < 2 \), and \( f(xf(y)) f(y) = f(x + y) \) for all \( x, y \).

6. Given a finite set of points in the plane, each with integer coordinates, is it always possible to color the points red or white so that for any straight line \( L \) parallel to one of the coordinate axes the difference (in absolute value) between the numbers of white and red points on \( L \) is not greater than 1?