Deformation Constraints in a Mass-Spring Model to Describe Rigid Cloth Behavior

Xavier Provot, 1995

Jeff Budsberg • April 12, 2005 • Cornell University
Early Work in Cloth

• Geometric models
  – Do not consider cloth’s physical properties
  – focus on appearance (particularly folds and creases)
  – Jerry Weil, 1986

• Physical models
  – Various structural studies are done and cloth’s intrinsic behavior is attempted to be simulated
  – C. Feynman, 1986
  – Demetri Terzopoulos et al, 1987
Early Work in Cloth

- Particle models
  - Explicitly represents the microstructure of woven cloth with interacting particles
  - David Breen et al., 1994
Jerry Weil

- Probably the first person to model cloth in any method whatsoever
- A cable under self-weight forms a catenary curve at equilibrium
- A cloth hanging from a discrete number of points can be described by a system of these curves

\[ y = a \cosh(x/b) \]
Jerry Weil

Surface Approximation

6 Iterations of Relaxation

Spline Fit
C. Feynman

- Represented cloth in a 3D space by using a 2D grid
- The energy for each point is calculated in relation to its surrounding points
- The final position of cloth was derived based on the minimization of energy

\[ E(P_{i,j}) = k_s E_{elastic(i,j)} + k_b E_{bending(i,j)} + k_g E_{gravitational(i,j)} \]
Demetri Terzopoulous

- Introduced a deformable model intended for generalized flexible objects
- Does not consider weave of the cloth, but only one internal elastic force
- Uses the Lagrange equation of motion to determine the equilibrium
Demetri Terzolpoulos
Motives

• Woven fabrics are far from having ideal elastic properties
• Physically-based, elastically-deformable models somewhat successful (“super-elastic” problem)
• Attempts at other methods:
  – Network of rigid rods of a fixed length (shearing, slow)
  – Particle system (static, slow)
Cloth Grid Mesh

Structural springs, resist stretching stresses
Sheer springs, resist sheering stresses
Flexion springs, resist bending stresses
Dynamics and Forces

- Once a mass-spring grid has been created, forces are applied to the nodes to generate an animation.
- The system is the mesh of \( m \times n \) masses, each mass with position at time \( t \) given by \( P_{i,j}(t) \).
- The evolution of the system is governed by the fundamental law of dynamics:

\[
F_{i,j} = \mu a_{i,j}
\]

where \( \mu \) is the mass at point \( P_{i,j}(t) \), and \( a_{i,j} \) is the acceleration caused by the force \( F_{i,j} \).
- \( F_{i,j} \) can be divided into internal and external forces.
Internal Force

• Tensions of interconnected springs

Which are described by Hooke’s Law:

\[ F = k \cdot u \]

where \( F \) is the applied force, \( u \) is the deformation (displacement from equilibrium) of the elastic body subjected to the force \( F \), and \( k \) is the spring constant.
Internal Force

- In our case, this force is basically the sum of the change in point vectors multiplied by the spring stiffness for each neighbor of each point.

\[
F_{int}(P_{i,j}) = - \sum_{(k,l) \in \mathcal{R}} K_{i,j,k,l} \left[ l_{i,j,k,l}^0 - l_{i,j,k,l}^0 \frac{l_{i,j,k,l}}{\|l_{i,j,k,l}\|} \right]
\]

- \(\mathcal{R}\) is the set regrouping all couples \((k,l)\) such as \(P_{k,l}\) is linked by a spring to \(P_{i,j}\),

- \(l_{i,j,k,l} = \overrightarrow{P_{i,j}P_{k,l}}\),

- \(l_{i,j,k,l}^0\) is the natural length of the spring linking \(P_{i,j}\) and \(P_{k,l}\),

- \(K_{i,j,k,l}\) is the stiffness of the spring linking \(P_{i,j}\) and \(P_{k,l}\).
External forces

• Force of gravity

\[ F_{gr}(P_{i,j}) = \mu g \]

where \( g \) is the acceleration of gravity

• Viscous damping

\[ F_{dis}(P_{i,j}) = -C_{dis}v_{i,j} \]

where \( C_{dis} \) is the damping coefficient and \( v_{i,j} \) is the velocity at point \( P_{i,j} \).
External forces

- **Viscous fluid (wind)**

\[
F_{vi}(P_{i,j}) = C_{vi} [n_{i,j} \cdot (u_{\text{fluid}} - v_{i,j})] n_{i,j}
\]

where \( u_{\text{fluid}} \) is a viscous fluid with uniform velocity, \( v_{i,j} \) is the velocity at point \( P_{i,j} \), \( n_{i,j} \) is the unit normal at \( P_{i,j} \), and \( C_{vi} \) is the viscosity constant.

- **The net force** acting on any node in the mass-spring model is the **sum of the above forces** for that node.
Integration

• To generate animation of cloth, it is necessary to compute the location of the nodes for a series of time steps.

• Provot uses a simple Euler method to approximate the fundamental equation of dynamics.

\[
\begin{align*}
    a_{i,j}(t + \Delta t) &= \frac{1}{\mu} F_{i,j}(t) \\
    v_{i,j}(t + \Delta t) &= v_{i,j}(t) + \Delta t \ a_{i,j}(t + \Delta t) \\
    P_{i,j}(t + \Delta t) &= P_{i,j}(t) + \Delta t \ v_{i,j}(t + \Delta t)
\end{align*}
\]
Forward Euler

- In this method, the position of the nodes in the next time step are computed using only past information.
Forward Euler Error

- Explicit integration has numerous problems including instability at large time steps and slow propagation of the effects of forces over the cloth material.
Dynamic Inverse Procedures

• Some cases where cloth movement is not entirely caused by analytically computed forces (contact problems)
• So far, we can compute displacement of a point due to a force applied to it, but we can solve the inverse problem for hanging points
• A similar procedure can be used to deal with object collisions and self-intersection, though not covered in this paper (Provot 97)
Collision Detection

• Collisions of two types
  – Point-triangle collision
  – Edge-edge collision
• At a time where a collision is detected, a physics-based response is calculated
• Accurate, but limited in that all nodes are assumed to have constant velocity
• Successful in simulating draping
• Problems with sliding contact and jittering
The “Super-Elastic” Effect

Initial position

After 200 iterations
The “Super-Elastic” Effect

- Case study: subject to gravity, but no wind
- Concentration of local deformations
- Deformation rate decreases very rapidly
- **Real-world problem:** such a deformation never occurs since real woven fabrics have non-linear elasticity (and tear when high loads are applied)

1) The deformation rate is defined as \( \tau = \frac{l - l_0}{l_0} \)
where \( l_0 \) is the natural spring length and \( l \) is its length at any time \( t \)
Increasing Stiffness

• Stiffer springs should lower the deformation rate
• For a given time step $\Delta t$ and mass $\mu$, there is a critical stiffness value $K_c$ above which the numerical resolution of the system is divergent
• Thus, the maximum $\Delta t$ is equal to the natural period of a simple harmonic oscillator (mass on a spring):

$$T_0 \approx \pi \sqrt{\frac{\mu}{K}}$$

$$\implies K_c \approx m \frac{T_0^2}{\pi^2}$$
Increasing Stiffness

- If we want to increase stiffness, we have to decrease $\Delta t$ below the new decreased value of $T_0$
- Need new method to avoid the superelastic effect, without decreasing $\Delta t$
Constraints on Deformation Rates

• Assume that the direction of the elongated spring is correct, but limit it to a critical deformation rate \( (\tau_c) \)

\[
\text{for } (\Delta t++) \\
\text{Compute every } \tau \\
\text{if } ( \epsilon \tau \| \tau > \tau_c ) \\
\tau = \tau_c
\]
Constraints on Deformation Rates

Adjustment of super-elongated spring linking two loose masses

Adjustment of super-elongated spring linking a fixed and a loose mass
Hanging Sheet Results

\[ r_c (\text{structural}) = 10\% \]
\[ r_c (\text{flexion}) = 0\% \]

\[ r_c (\text{structural/shear}) = 10\% \]
\[ r_c (\text{flexion}) = 0\% \]
Hanging Sheet Comparison

On the left: a stiff elastic model computed in 9 min.
On the right: new model computed in 1 min.
Flag in a Strong Wing

Semi-rigid flag and elastic flag when stiffness is low

Semi-rigid flag remains stable when stiffness is increased
Wind in a Sail

Hangs by 8 points on the upper rod and is ties to 2 points on the lower rod
Disadvantages

• Diverges from strictly physics-based simulation
• Dependent on the order in which the springs are examined
• Correcting one spring may overextend another
• Reiterative process does not always converge to a completely non-extended state
Advantages

• Produces realistic-looking output in most cases
• The time constant does not need to be reduced to match higher spring constants
• Can use an order of magnitude large time step
• 90% reduction in running time of the simulation according to the author
• Thus this model sacrifices a tolerable amount of accuracy for a dramatic speed improvement
Summary

- A physically-based model for animating cloth objects
- Derived from elastically-deforming models, but takes into account non-elastic properties of woven fabrics
- Cloth object approximated with a deformable surface network of masses and springs
- Dynamic inverse procedure to correct for unrealistic local deformation about the boundary conditions.