**Radiosity+Importance**

- Radiosity+Importance: Bidirectional

**Importance Radiosity (IR)**

- **Motivation**
  - $O(k^2 + n)$ is too slow
  - HR oversolves globally, undersolves locally
- **Insight**: Exploit view dependence
- **Importance**: Direct or indirect contribution of patch to image from this viewpoint

**IR Intuition**

- Importance: Adjoint formulation of radiosity

**IR Algorithm**

- Solves for dual system simultaneously
- Importance is shot by treating eye as light source
- Importance $R_i$ proportional $A_i$ on image
Radiosity Equation

- Radiosity for each polygon $i$
  \[ B_i = B_{e,i} + \rho_i \sum_{j \neq i} B_j F(i \rightarrow j) \]
- Linear system
  - $B_i$: radiosity of patch $i$ (unknown)
  - $B_{e,i}$: emission of patch $i$ (known)
  - $\rho_i$: reflectivity of patch $i$ (known)
  - $F(i \rightarrow j)$: form-factor (coefficients of matrix)

IR Intuition

- $I_1 = R_1 + \rho_2 F_{21} I_2 + \rho_3 F_{31} I_3 + …$

Importance Radiosity

- Elegant formulation of bidirectional propagation
  - Replaces ad-hoc solutions
- IR restricted to one viewpoint
  - Need to unmesh as viewpoint moves

Motivation

- Eye
- Scene
- ???

What is the behavior of light?

- Physics of light
- Radiometry
- Material properties

Models of Light

- Geometric Optics
  - Emission
  - Reflection / Refraction
  - Absorption
- Simplest model
- Size of objects $>$ wavelength of light
Radiometry

- Radiometry: measurement of light energy
- Defines relation between
  - Power
  - Energy
  - Radiance
  - Radiosity

Digression: Hemispheres

- Hemisphere = two-dimensional surface
- Direction = point on (unit) sphere

\[
\theta \in [0, \frac{\pi}{2}], \quad \varphi \in [0, 2\pi]
\]

Digression: Solid angles

2D

\[ \theta = \frac{L}{R} \]

3D

\[ \Omega = \frac{A}{R^2} \]

Full circle = \(2\pi\) radians
Full sphere = \(4\pi\) steradians

Digression: Solid angle

- Full sphere = \(4\pi\) steradian = 12.566 sr
- Dodecahedron = 12-sided regular polyhedron; 1 face = 1 sr

Hemispherical coordinates

- Direction = point on (unit) sphere

\[
dA = (r \sin \theta d\varphi) (r d\theta)
\]
Hemispherical coordinates

- Defined a measure over hemisphere
- \( d\omega = \text{direction vector} \)
- Differential solid angle

\[
d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi
\]

Hemispherical integration

- Area of hemisphere:

\[
\int d\omega = \int d\varphi \int_0^\pi \sin \theta d\theta
\]

\[
= \int_0^{2\pi} d\varphi [\cos \theta]^{1/2} d\theta
\]

\[
= \int_0^{2\pi} d\varphi
\]

\[
= 2\pi
\]

Power

- Energy: Symbol: \( Q \); unit: Joules
- Power: Energy per unit time \((dQ/dt)\)
  - Aka. “radiant flux” in this context
- Symbol: \( P \) or \( \Phi \); unit: Watts \((\text{Joules} / \text{sec})\)
  - Photons per second
  - All further quantities are derivatives of \( P \) (flux densities)

Irradiance

- Power per unit area \((dP/dA)\)
  - That is, area density of power
  - It is defined with respect to a surface
- Symbol: \( E \); unit: \( W / m^2 \)
  - Measurable as power on a small-area detector
  - Area power density exiting a surface is called radiant exitance \((M)\) or radiosity \((B)\) but has the same units

Irradiance example

- Uniform point source illuminates small surface \( dA \) from distance \( r \)
  - Think of it as a piece of a sphere
  - Power \( P \) is uniformly spread over the area of the sphere

\[
dP = P \frac{dA}{4\pi r^2}; E = \frac{dP}{dA} = \frac{P}{4\pi r^2}
\]

\[
E' = \frac{dP}{dA'} = \frac{dP}{dA' \cos \theta} = E \cos \theta
\]
Radiance

- Radiance is radiant energy at x in direction θ: 5D function
  - \( L(x \rightarrow \Theta) \): Power
    - per unit projected surface area
    - per unit solid angle
    \[
    L(x \rightarrow \Theta) = \frac{d^2P}{dA^\perp dA_o}
    \]
  - units: Watt / m².sr

- Why per unit projected surface area?

Radiance: Projected area

- \( L(x \rightarrow \Theta) = \frac{d^2P}{dA^\perp dA_o} \)

Why is radiance important?

- Invariant along a straight line (in vacuum)

Invariance of Radiance

- \( L(x \rightarrow y) = L(y \leftarrow x) \)

Why is radiance important?

- Response of a sensor (camera, human eye) is proportional to radiance
- Pixel values in image proportional to radiance received from that direction
Wavelength Dependence

- Each particle has a wavelength: \( E = \frac{h}{\lambda} \)
- All radiometric quantities depend on wavelength
- Spectral radiance: \( L(x \to \Theta, \lambda) \)
- Radiance: \( L(x \to \Theta) = \int L(x \to \Theta, \lambda) d\lambda \)

Relationships

- Radiance is the fundamental quantity
  \[ L(x \to \Theta) = \frac{d^2 P}{dA d\omega} \]
- Power:
  \[ P = \int_{A_s, \text{Solid Angle}} L(x \to \Theta) \cdot \cos \theta \cdot d\omega \cdot dA \]
- Radiosity:
  \[ B = \int_{A_s, \text{Solid Angle}} L(x \to \Theta) \cdot \cos \theta \cdot d\omega \]

Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere
  \[ L(x \to \Theta) = \frac{d^2 P}{dA d\omega} \]
  \[ P = \int_{A_s, \text{Solid Angle}} L(x \to \Theta) \cdot \cos \theta \cdot d\omega \cdot dA \]
  \[ = L \int_{A_s} dA \int \cos \theta \cdot d\omega \]
  \[ = L \cdot A_s \cdot \pi \]

Sun Example: radiance

- Power: \( 3.91 \times 10^{26} \) W
- Surface Area: \( 6.07 \times 10^{18} \) m\(^2\)
- Power = Radiance \cdot Surface Area \cdot \pi
- Radiance = Power / (Surface Area \cdot \pi)
- Radiance = \( 2.05 \times 10^7 \) W/m\(^2\)sr

Sun Example: Power on Earth

- Power reaching earth on a 1m\(^2\) square:
  \[ P = L \int_{A_s} dA \int \cos \theta \cdot d\omega \]
- Assume \( \cos \theta = 1 \) (sun in zenith)
  \[ P = L \int_{A_s} dA \int d\omega \]

Same radiance on Earth and Mars?
Sun Example: Power on Earth

Power = Radiance. Area. Solid Angle

Solid Angle = Projected Area_{sun} / (distance_{earth\_sun})^2
= 6.7 \times 10^{-5} \text{ sr}

P = (2.05 \times 10^7 \text{ W/m}^2\cdot\text{sr}) \times (1 \text{ m}^2) \times (6.7 \times 10^{-5} \text{ sr})
= 1373.5 \text{ Watt}

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Sun Example: Power on Mars

Power = Radiance. Area. Solid Angle

Solid Angle = Projected Area_{sun} / (distance_{mars\_sun})^2
= 2.92 \times 10^{-5} \text{ sr}

P = (2.05 \times 10^7 \text{ W/m}^2\cdot\text{sr}) \times (1 \text{ m}^2) \times (2.92 \times 10^{-5} \text{ sr})
= 598.6 \text{ Watt}

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