Problem Set 7

Due Date: Thurs, March 13, 2003

Reading

Please read the handout on second order propositional logic (on either the 2001 or 2003 CS486 web site); Smullyan, Chapter IV, p. 43-52; and Suppes’ Introduction to Logic, pages 43-54.

Problems


2. Prove or disprove these $P^2$ formulas:
   
   (a) $\forall p \forall q ((p \supset q) \supset ((p \supset \bot) \supset (q \supset \bot)))$
   (b) $\forall p \exists q ((p \supset q) \supset ((p \supset \bot) \supset (q \supset \bot)))$
   (c) $(\forall p(\sim p)) \supset \sim \exists p.p$
   (d) $\forall p \exists q ((p \lor q) \supset p)$
   (e) $\forall p \exists q ((p \lor q) \land \sim (p \land q))$

3. Reduce these $P^2$ formulas to $P^0$ formulas.

   (a) $\forall p(p) \supset \bot$
   (b) $\forall p \forall q ((\sim p \lor q) \supset (p \supset q))$
   (c) $\forall p \forall q ((p \supset p \lor q) \land (p \land q \supset p))$


5. Solve exercise 3, page 52 of Smullyan.


7. There is a simple proof for cut elimination in $P^2$. State the theorem and outline a proof. Details are not necessary.

Optional Problem:

Develop the idea that Refinement Logic is a calculus of “problems.” Try to define the idea of a problem $P$ and a solution for it, say $s$. Use a semantics based on the idea that $s$ solves $P$. 

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