1. Prove that $1 + x \leq e^x$ for all real $x$. For what values of $x$ is the approximation $1 + x = e^x$ good?

2. Let $f(n)$ be a function that is asymptotically less than $n$. Some such functions are $1/n$, a constant $d$, $\log n$ or $n^{\frac{1}{2}}$. Show that

$$\lim_{n \to \infty} \left(1 + \frac{f(n)}{n}\right)^n = e^{f(n)}.$$

3. Let $G(n, p)$ be a random graph and let $x$ be the random variable denoting the number of unordered pairs of non-adjacent vertices $(u, v)$ such that no other vertex of $G$ is adjacent to both $u$ and $v$. Prove that if $\lim E(x) = 0$, then for large $n$ there are almost no disconnected graphs i.e. $\text{Prob}(x = 0) \to 1$ and hence $\text{Prob}(G \text{ is connected}) \to 1$. Actually the graph becomes connected long before this condition is true.

4. Let $x_i$, $1 \leq i \leq n$, be a set of indicator variables with identical probability distributions. Let $x = \sum_{i=1}^{n} x_i$ and suppose $E(x) = \infty$. Show that if the $x_i$ are statistically independent, then $\text{Prob}(x = 0) = 0$.

5. In the proof that every monotone property has a threshold we cannot say that $G(n, q)$ has the property $Q$ only if one of the $G(n, p(\varepsilon))$ has the property $Q$ even though $G(n, q)$ is the union of the $G(n, p(\varepsilon))$. $G(n, q)$ might have the property even though none of the $G(n, p(\varepsilon))$ have the property. Give an example of such a property.

6. Consider a model of a random subset $N(n, p)$ of integers $\{1, 2, \ldots n\}$ where, $N(n, p)$ is the set obtained by independently at random including each of $\{1, 2, \ldots n\}$ into the set with probability $p$. What is the threshold for $N(n, p)$ to contain a) a perfect square, b) a perfect cube, c) an even number, d) three numbers such that $x+y=z$. 

Homework assignment 3 due Friday February 19
To expedite grading please submit each problem on a separate sheet of paper.