Motivation for generative models

- Word prediction
  - Once upon a...
  - I'd like to make a collect...
  - Let's go outside and take a...
- The need for models of word prediction in NLP has not been uncontroversial
  - But it must be recognized that the notion "probability of a sentence" is an entirely useless one, under any known interpretation of this term. - Noam Chomsky (1969)
  - Every time I fire a linguist the recognition rate improves. - Fred Jelinek (IBM speech group, 1988)

N-gram model

- Uses the previous N-1 words to predict the next one
  - 2-gram: bigram
  - 3-gram: trigram
  - 1-gram: unigram
- In speech recognition, these statistical models of word sequences are referred to as a language model

Models of word sequences

- Simplest model
  - Let any word follow any other word
    » P (word1 follows word2) = 1/# words in English
- Probability distribution at least obeys actual relative word frequencies
  » P (word1 follows word2) =
    # occurrences of word1 / # words in corpus
- Pay attention to the preceding words
  - “Let’s go outside and take a [ ]”
    » walk very reasonable
    » break quite reasonable
    » shower less reasonable
  - Compute conditional probability  P (walk | let’s go…)
Probability of a word sequence

- \( P(w_1, w_2, \ldots, w_{n-1}, w_n) \)

\[
P(w_n^n) = P(w_1) \cdot P(w_2 | w_1) \cdot P(w_3 | w_2) \cdots P(w_n | w_{n-1}^{n-1})
= \prod_{k=1}^{n} P(w_k | w_{k-1}^{k-1})
\]

Problem?

Solution: approximate the probability of a word given all the previous words...

N-gram approximations

- Bigram model
  \[
P(w_n | w_{n-1}^{n-1}) \approx P(w_n | w_{n-1})
\]
  \[
P(w_n^n) \approx \prod_{k=1}^{n} P(w_k | w_{k-1}^{k-1})
\]
- Trigram model
  - Conditions on the two preceding words

N-gram approximation

\[
P(w_n^n) \approx \prod_{k=1}^{n} P(w_k | w_{k-N-1}^{k-1})
\]

Markov assumption: probability of some future event (next word) depends only on a limited history of preceding events (previous words)

Bigram grammar fragment

- Berkeley Restaurant Project

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>eat on</td>
<td>.16</td>
<td>eat Thai</td>
</tr>
<tr>
<td>eat some</td>
<td>.06</td>
<td>eat break</td>
</tr>
<tr>
<td>eat lunch</td>
<td>.06</td>
<td>eat in</td>
</tr>
<tr>
<td>eat dinner</td>
<td>.05</td>
<td>eat Chinese</td>
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<tr>
<td>eat at</td>
<td>.04</td>
<td>eat Mexican</td>
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<tr>
<td>eat a</td>
<td>.04</td>
<td>eat tomorrow</td>
</tr>
<tr>
<td>eat Indian</td>
<td>.04</td>
<td>eat dessert</td>
</tr>
<tr>
<td>eat today</td>
<td>.03</td>
<td>eat British</td>
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</table>

Can compute the probability of a complete string
- \( P(I \text{ want to eat British food}) = P(I|<s>) \cdot P(\text{want}|I) \cdot P(\text{to}|\text{want}) \cdot P(\text{eat}|\text{to}) \cdot P(\text{British}|\text{eat}) \cdot P(\text{food}|\text{British}) \)

Training N-gram models

- N-gram models can be trained by counting and normalizing
  - Bigrams
    \[
P(w_n | w_{n-1}) = \frac{C(w_{n-1}^{n-1}w_n)}{C(w_{n-1})}
\]
  - General case
    \[
P(w_n | w_{n-N+1}^{n-1}) = \frac{C(w_{n-N+1}^{n-1}w_n)}{C(w_{n-N+1}^{n-1})}
\]
  - An example of Maximum Likelihood Estimation (MLE)
    » Resulting parameter set is one in which the likelihood of the training set \( T \) given the model \( M \) (i.e. \( P(T|M) \)) is maximized.
Bigram counts

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- Note the number of 0's...

Bigram probabilities

- Problem for the maximum likelihood estimates: sparse data

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</tr>
</tbody>
</table>

Accuracy of N-gram models

- Accuracy increases as N increases
  - Train various N-gram models and then use each to generate random sentences.
  - Corpus: Complete works of Shakespeare
    - Unigram: *Will rash been and by I the me loves gentle me not slavish page, the and hour; ill let*
    - Bigram: *What means, sir. I confess she? Then all sorts, he is trim, captain.*
    - Trigram: *Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.*
    - Quadrigram: *They say all lovers swear more performance than they are wont to keep obliged faith unforfeited!*

Strong dependency on training data

- Trigram model from WSJ corpus
  - *They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions*