**Parametric surfaces and solid modeling**

CS 465 Lecture 12

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**From curves to surfaces**

- So far have discussed spline curves in 2D
  - it turns out that this already provides the mathematical machinery for several ways of building curved surfaces
- Building surfaces from 2D curves
  - extrusions and surfaces of revolution
- Building surfaces from 2D and 3D curves
  - generalized swept surfaces
- Building surfaces from spline patches
  - generalizing spline curves to spline patches
- Also to think about: generating triangles

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**Extrusions**

- Given a spline curve $C \in \mathbb{R}^2$, define $S \in \mathbb{R}^3$ by $S = C \times [a, b]$
- This produces a “tube” with the given cross section
  - Circle: cylinder; “L”: shelf bracket; “I”: I beam
- It is parameterized by the spline $t$ and the interval $[a, b]$
  $$s(t, s) = [c_x(t), c_y(t), s]^T$$

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**Surfaces of revolution**

- Take a 2D curve and spin it around an axis
- Given curve $\mathbf{c}(t)$ in the plane, the surface is defined easily in cylindrical coordinates:
  $$s(t, s) = (r, \phi, z) = (c_x(t), s, c_y(t))$$
  - the torus is an example in which the curve $\mathbf{c}$ is a circle
**Swept surfaces**

- Surface defined by a cross section moving along a spine
- Simple version: a single 3D curve for spine and a single 2D curve for the cross section

**Generalized cylinders**

- General swept surfaces
  - varying radius
  - varying cross-section
  - curved axis

**From curves to surface patches**

- Curve was sum of weighted 1D basis functions
- Surface is sum of weighted 2D basis functions
  - construct them as separable products of 1D fns.
  - choice of different splines
    - spline type
    - order
    - closed/open (B-spline)

**Separable product construction**
Bilinear patch

- Simplest case: 4 points, cross product of two linear segments
  - basis function is a 3D tent

Bicubic Bézier patch

- Cross product of two cubic Bézier segments
  - properties that carry over
    - interpolation at corners, edges
    - tangency at corners, edges
    - convex hull

Biquadratic Bézier patch

- Cross product of quadratic Bézier curves

3x5 Bézier patch

- Cross product of quadratic and quartic Béziers
Cylindrical B-spline surfaces

- Cross product of closed and open cubic B-splines

Approximating spline surfaces

- Similarly to curves, approximate with simple primitives
  - in surface case, triangles or quads
  - quads widely used because they fit in parameter space
    - generally eventually rendered as pairs of triangles
- adaptive subdivision
  - basic approach: recursively test flatness
    - if the patch is not flat enough, subdivide into four using
      curve subdivision twice, and recursively process each
      piece
  - as with curves, convex hull property is useful for
    termination testing (and is inherited from the curves)

Approximating spline surfaces

- With adaptive subdivision, must take care with cracks
  - (at the boundaries between degrees of subdivision)

Modeling in 3D

- Representing subsets of 3D space
  - volumes (3D subsets)
  - surfaces (2D subsets)
  - curves (1D subsets)
  - points (0D subsets)
Representing geometry

- In order of dimension...
- Points: trivial case
- Curves
  - normally use parametric representation
  - line—just a point and a vector (like ray in ray tracer)
    - polylines (approximation scheme for drawing)
  - more general curves: usually use splines
    - \( p(t) \) is from \( \mathbb{R} \) to \( \mathbb{R}^3 \)
    - \( p \) is defined by piecewise polynomial functions

Representing geometry

- Surfaces
  - this case starts to get interesting
  - implicit and parametric representations both useful
    - example: plane
      - implicit: vector from point perpendicular to normal
      - parametric: point plus scaled tangent
    - example: sphere
      - implicit: distance from center equals \( r \)
      - parametric: write out in spherical coordinates
        - messiness of parametric form not unusual

Representing geometry

- Volumes
  - boundary representations (B-reps)
    - just represent the boundary surface
    - convenient for many applications
    - must be closed (watertight) to be meaningful
      - an important constraint to maintain in many applications

Representing geometry

- Volumes
  - CSG (Constructive Solid Geometry)
    - apply boolean operations on solids
    - simple to define
    - simple to compute in some cases
      - [e.g. ray tracing]
    - difficult to compute stably with B-reps
      - [e.g. coincident surfaces]
Specific surface representations

- Parametric surfaces
  - extrusions
  - surfaces of revolution
  - generalized cylinders
  - spline patches

Specific surface rep.

- Algebraic implicit surfaces
  - defined as zero sets of fairly arbitrary functions
  - good news: CSG is easy using min/max
  - bad news: rendering is tough
    - ray tracing: intersect arbitrary zero sets w/ray
    - pipeline: need to convert to triangles
      - e.g. “blobby” modeling

Specific surface representations

- Isosurface of volume data
  - implicit representation
  - function defined by regular samples on a 3D grid
    - (like an image but in 3D)
  - example uses:
    - medical imaging
    - numerical simulation

Specific surface representations

- Triangle or polygon meshes
  - parametric (per face)
  - very widely used
    - final representation for pipeline rendering
    - these days restricting to triangles is common
  - rather unstructured
    - need to be careful to enforce necessary constraints
    - to bound a volume need a watertight manifold mesh
Specific surface representations

- Subdivision surfaces
  - based on polygon meshes (quads or triangles)
  - rules for subdividing surface by adding new vertices
  - converges to continuous limit surface

Subdivision of meshes

- Quadrilaterals
  - Catmull-Clark 1978
- Triangles
  - Loop 1987

Loop regular rules

Loop Subdivision Example

color polyhedron
Loop Subdivision Example

refined control polyhedron

odd subdivision mask

subdivision level 1

even subdivision mask (ordinary vertex)
Loop Subdivision Example

subdivision level 1

Loop Subdivision Example

subdivision level 2

Loop Subdivision Example

subdivision level 3
Loop Subdivision Example

subdivision level 4

limit surface