Problem 1: Splines (25 pts)

(a) Complete the table below by indicating which features each of the following types of spline is guaranteed to preserve in a spline that is many segments long. Assume that for Catmull-Rom we duplicate the two endpoints of the set of control points when generating the curve.

<table>
<thead>
<tr>
<th>Type</th>
<th>$C^1$</th>
<th>$C^2$</th>
<th>Interpolates endpoints</th>
<th>Contained within convex hull</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catmull-Rom</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bézier</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cubic B-spline</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Suppose we create a Catmull-Rom curve, a Bézier curve, and a Cubic B-spline curve from the four control points show below. Again, assume that for Catmull-Rom we duplicate the two endpoints (points 0 and 3) when generating the curve. Order the three curves from the highest left endpoint to the lowest left endpoint (measured on the y-axis). If there are ties, break them by ordering from the highest midpoint to the lowest midpoint among all those with the same endpoint height.
Problem 2: Viewing (25 pts)

Match each matrix with the single classification that best describes the role that matrix most likely plays as part of the graphics pipeline (each matrix should be matched with a single classification, and vice versa).

(1) Modeling transformation matrix  (2) Viewport matrix
(3) Viewing matrix (camera frame matrix)  (4) Projection matrix

(a) \[
\begin{bmatrix}
\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -\frac{3\sqrt{2}}{2} \\
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
(b) \[
\begin{bmatrix}
-\frac{10}{7} & 0 & -\frac{1}{7} & 0 \\
0 & -\frac{5}{3} & -\frac{1}{3} & 0 \\
0 & 0 & -\frac{25}{2} & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]
(c) \[
\begin{bmatrix}
320 & 0 & 0 & \frac{639}{2} \\
0 & 240 & 0 & \frac{479}{2} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
(d) \[
\begin{bmatrix}
2 & 0 & 0 & -8 \\
0 & \frac{3}{2} & \frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{4} - 3 \\
0 & -\frac{3\sqrt{3}}{2} & -\frac{1}{4} & -3\sqrt{3} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Problem 3: Pipeline (25 pts)

(a) For each stage below, briefly (1-2 sentences) explain its role in the graphics pipeline and what it is responsible for producing for the next stage

i. Geometry Processing
ii. Rasterization
iii. Fragment Processing

(b) Consider the following types of rendering. For each one, list the attributes that would need to be interpolated during rasterization. Assume an infinite viewer and infinite light unless otherwise specified.

i. Phong lighting with Gouraud shading and local (non-directional) lights
ii. Phong lighting with Phong shading and local viewer
iii. Texture mapping
Problem 4: Meshes (25 pts)

(a) Give a representation of the mesh below as

i. An indexed triangle mesh
ii. A set of triangle strips (using as few strips as possible)

(b) Suppose you want to perform the following queries on some given triangle mesh. For each query, list all of the storage formats (indexed triangle mesh, triangle strips, and winged-edge) that allow the query to be done efficiently, where efficient means independent of the total size of the mesh.

i. Find all the vertices for a given triangle.
ii. Find all the adjacent triangles for a given triangle.
iii. Find all the triangles around for a given vertex.
iv. Find all the adjacent vertices for a given vertex.