CS4620/5620: Lecture 30

Splines and Animation

Announcements

• 4621
  – Friday (Animation)

• HW 3 released tonight
Beziers curve
Recursive algorithm: Option 2

void DrawRecBezier (float eps) {
  if Linear (curve, eps)
    DrawLine (curve);
  else
    SubdivideCurve (curve, leftC, rightC);
    DrawRecBezier (leftC, eps);
    DrawRecBezier (rightC, eps);
}

Termination Criteria

• Test for linearity
  – distance between control points
  – distance of control points from line

\[ d_0 < \varepsilon \]
\[ d_1 < \varepsilon \]
B-splines

• We may want more continuity than $C^1$

• B-splines are a clean, flexible way of making long splines with arbitrary order of continuity

• Various ways to think of construction
  – a simple one is convolution

• An approximating spline

Deriving the B-Spline

• Approached from a different tack than Hermite-style constraints
  – Want a cubic spline; therefore 4 active control points
  – Want $C^2$ continuity
  – Turns out that is enough to determine everything
Efficient construction of any B-spline

• B-splines defined for all orders
  – order $d$: degree $d - 1$
  – order $d$: $d$ points contribute to value
• One definition: Cox-deBoor recurrence

\[
b_1 = \begin{cases} 
1 & 0 \leq u < 1 \\
0 & \text{otherwise}
\end{cases}
\]

\[
b_d = \frac{t}{d-1} b_{d-1}(t) + \frac{d-t}{d-1} b_{d-1}(t-1)
\]

B-spline construction, alternate view

• Recurrence
  – ramp up/down

• Convolution
  – smoothing of basis fn
  – smoothing of curve

\[\text{Graphs of } b_1(t), b_2(t), b_3(t), b_4(t), b_5(t)\]
Cubic B-spline matrix

\[ p(t) = \begin{bmatrix} t^3 & t^2 & t \end{bmatrix} \cdot \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_{k-1} \\ p_k \\ p_{k+1} \\ p_{k+2} \end{bmatrix} \]

Cubic B-spline curves

- Treat points uniformly
- C^2 continuity
- \( C(t) = [(1-t)^3 P_{i-3} + (3t^3 - 6 t^2 + 4)P_{i-2} + (-3 t^3 + 3t^2 + 3t + 1) P_{i-1} + t^3 P_{i}] / 6 \)
- Notice blending functions still add to 1
Cubic B-spline basis

- B-spline from each 4-point sequence matches previous, next sequence with $C^2$ continuity!
- Treats all points uniformly
Rendering the spline-curve

• Given B-spline points \( d_{-1}, \ldots d_{L+1} \)
• Compute Bézier points \( b_0, \ldots b_{3L} \)
• Use De Casteljau algorithm to render

![Diagram showing the process of rendering a spline curve using De Casteljau algorithm.]

Equations and boundary conditions

• Equations
  • \( b_{3i} = (b_{3i-1} + b_{3i+1}) / 2 \)
  • \( b_{3i-1} = d_{i-1}/3 + 2d_i/3 \)
  • \( b_{3i-2} = 2d_{i-1}/3 + d_i/3 \)

• Boundary conditions
  • \( b_0 = d_{-1}, b_1 = d_0, b_2 = (d_0 + d_1)/2 \)
  • \( b_{3L} = d_{L+1}, b_{3L-1} = d_L, b_{3L-2} = (d_{L-1} + d_L)/2 \)
Converting spline representations

• All the splines we have seen so far are equivalent
  – all represented by geometry matrices
  \[ p_S(t) = T(t)M_SP_S \]
  
  • where \( S \) represents the type of spline
  – therefore the control points may be transformed from one type to another using matrix multiplication
  \[ P_1 = M_1^{-1}M_2P_2 \]
  \[ p_1(t) = T(t)M_1(M_1^{-1}M_2P_2) = T(t)M_2P_2 = p_2(t) \]

Other types of B-splines

• Nonuniform B-splines
  – discontinuities not evenly spaced
  – allows control over continuity or interpolation at certain points
  – e.g. interpolate endpoints (commonly used case)
• Nonuniform Rational B-splines (NURBS)
  – ratios of nonuniform B-splines: \( x(t)/w(t), \ y(t)/w(t) \)
  – key properties:
    • invariance under perspective
    • ability to represent conic sections exactly
Why Splines?

• Advantages
  • Smooth curves: $C^0$, $C^1$, $C^2$ continuity
  • Intuitive editing
  • Smooth animation
  • Compact: represent complex curves simply
  • Convenient parametric representation
    • Point $p = (x,y) = (x(t), y(t))$

Surfaces

• Generalize by product of basis functions in 2 dimensions
Summary

- Splines
  - Smoothness, continuity (C0, C1, C2)
- Hermite
  - No convex hull property
  - 2 points and 2 tangents
- Bezier
  - Convex hull property
  - De Casteljau evaluation
  - Invariant to affine transformations
- B splines
  - Non-interpolation; i.e., approximating splines
  - C2 continuity

Animation
Animation

- Industry production process leading up to animation
- How animation works (very generally)
- Artistic process of animation
- Further topics in how it works

What is animation?

- Modeling = specifying shape
- Animation = specifying shape as a function of time
  - Just modeling done once per frame?
  - Need smooth, concerted movement
- Controlling shape = the technical problem
- Using shape controls = the artistic problem
Approaches to animation

• Straight ahead
  – Draw/animate one frame at a time
  – Can lead to spontaneity, but is hard to get exactly what you want

• Pose-to-pose
  – Top-down process:
    • Plan shots using storyboards
    • Plan key poses first
    • Finally fill in the in-between frames
Pose-to-pose animation planning

– First work out poses that are key to the story
– Next fill in animation in between

Keyframe animation

• Keyframing is the technique used for pose-to-pose animation
  – Head animator draws key poses—just enough to indicate what the motion is supposed to be
  – Assistants do “in-betweening” and draw the rest of the frames
  – In computer animation substitute “user” and “animation software”
    – Interpolation is the main operation
  – Pro: lots of artistic control
  – Con: Manually intensive
Principles of Animation

- Classic paper by Lasseter

Principles of Animation

- Timing
- Ease In and Out (or Slow In and Out)
- Arcs
- Anticipation
- Exaggeration
- Squash and Stretch
- Secondary Action
- Follow Through and Overlapping Action
- Straight Ahead Action and Pose-To-Pose Action
- Staging
- Appeal
- Personality
Animation principles: timing

- Speed of an action is crucial to the impression it makes
  - gives physical and emotional meaning
  - examples with same keyframes, different times:

60 fr: looking around  
30 fr: “no”  
5 fr: just been hit

Timing

- Indicates emotional state
- Eg. Look over left shoulder, then right

- On a scale of 1 to 10
  No in-between: snap
  1 in-between: hit with force
  2 in-betweens: nervous twitch
  3 in-betweens: dodging something
  4 in-betweens: giving an order
  6 in-betweens: sees something inviting
  9 in-betweens: thinking
  10 in-betweens: stretching
Animation principles: ease in/out

- Real objects do not start and stop suddenly
  - animation parameters shouldn’t either

- a little goes a long way (just a few frames acceleration or deceleration for “snappy” motions)

Animation principles: moving in arcs

- Real objects also don’t move in straight lines
  - generally curves are more graceful and realistic
**Animation principles: anticipation**

- Most actions are preceded by some kind of “wind-up”

![Animation principle: anticipation](image1)

**Animation principles: exaggeration**

- Animation is not about exactly modeling reality
- Exaggeration is very often used for emphasis

![Animation principle: exaggeration](image2)
Animation principles: squash & stretch

• Objects do not remain perfectly rigid as they move
• Adding stretch with motion and squash with impact:
  – models deformation of soft objects
Animation principles: follow through

- We've seen that objects don't start suddenly
- They also don't stop on a dime
  - Let arm complete motion
  - Let leg kick complete motion

Anim. principles: overlapping action

- Usually many actions are happening at once

- Have a plan
Principles of Animation

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Animation principles: staging

- Want to produce clear, good-looking 2D images
- Attract attention to key character/actor
  - need good camera angles, set design, and character positions
  - rim lighting
Principles at work: weight

Computer-generated motion

- Interesting aside: many principles of character animation follow indirectly from physics
- Anticipation, follow-through, and many other effects can be produced by simply minimizing physical energy
- Seminal paper: “Spacetime Constraints” by Witkin and Kass in SIGGRAPH 1988